

MECÂNICA COMPUTACIONAL
Licenciatura em Engenharia Biomédica

1ª Época

Ano Lectivo de 2003/2004

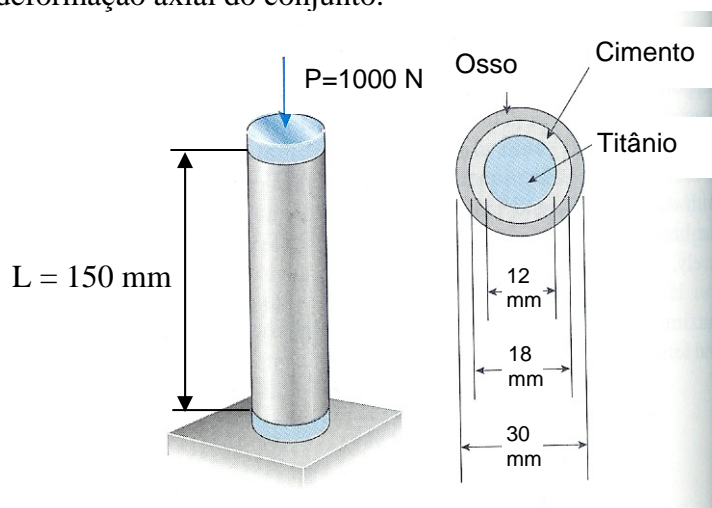
17/Junho/2004

- O Exame é sem consulta. O formulário está anexo a este exame.
- Não são permitidos computadores pessoais ou telemóveis.
- Todas as folhas do exame deverão ser identificadas.
- A duração do exame é de 2 horas e 30 minutos.

Problema I (5 val.)

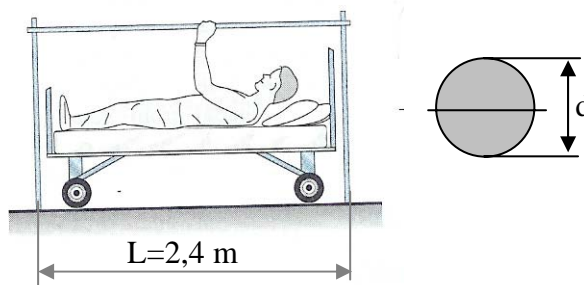
A figura representa uma parte da diáfise do fémur onde se implantou uma prótese cimentada de secção circular. Admita que o esforço predominante é axial e corresponde a uma força de 1000 N aplicada como mostra a figura. Considere o módulo de Young para o osso de 20 GPa, para o cimento de 2,5 GPa e para o Titânio de 110 GPa.

- a) Determine as tensões na prótese, no cimento e no osso compacto.
- b) Calcule a deformação axial do conjunto.



Problema II (4 val.)

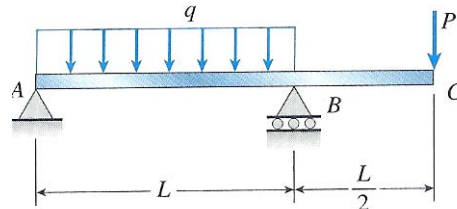
Uma “barra de trapézio” numa sala de hospital proporciona um meio de exercício a doentes acamados (ver figura). A barra tem 2,4 metros de comprimento e uma secção transversal circular. Considerando que a carga aplicada é de 1100 N no ponto médio da “barra”, determine o diâmetro da barra para uma tensão admissível de 180 MPa. Admita que a “barra” está simplesmente apoiada nas duas extremidades.



Problema III (6 val.)

Considere a viga em aço ($E= 200 \text{ GPa}$) representada na figura. O comprimento entre apoios é $L=10$ metros e a secção da viga é rectangular com $I=1.8 \times 10^{-3} \text{ m}^4$. O carregamento distribuído é de $q=400 \text{ N/m}$ e a carga concentrada $P= 10 \text{ kN}$.

- Desenhe os diagramas de esforço transversal e momento flector, e indique os respectivos valores máximos absolutos.
- Calcule a equação da curva elástica da viga.
- Determine a deformada no ponto C.



Problema IV (5 val.)

Pretende-se resolver o problema III utilizando o método dos elementos finitos.

- Indique justificando que tipo de elemento utilizaria e qual o menor número de elementos necessário para resolver o problema.
- Determine, sem fazer cálculos, o vector de cargas global para a malha escolhida em a). Indique detalhadamente todos os cálculos que eventualmente teria de fazer.
- Comente a precisão da solução obtida para o diagrama de momento flector pelo Método dos Elementos Finitos.

FORMULÁRIO:

$$\sigma = \frac{P}{A} \quad \sigma = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A} \quad \tau = \lim_{\Delta A \rightarrow 0} \frac{\Delta V}{\Delta A} \quad \tau_{xy} = \tau_{yx} \quad \sigma = \frac{P}{A_0} \cos^2 \theta$$

$$\tau = \frac{P}{A_0} \cos \theta \sin \theta \quad \varepsilon = \frac{d\delta}{dx} \quad \sigma = E\varepsilon \quad \varepsilon_T = \alpha \Delta T \quad \delta = \frac{PL}{AE}$$

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i} \quad \delta = \int_0^L \frac{P}{AE} dx$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E} \quad \varepsilon_y = -\nu \frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E} \quad \varepsilon_z = -\nu \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$\tau_{xy} = G\gamma_{xy} \quad \tau_{yz} = G\gamma_{yz} \quad \tau_{xz} = G\gamma_{xz} \quad G = \frac{E}{2(1+\nu)}$$

$$\gamma = \frac{\rho\phi}{L} \quad \tau = \frac{T\rho}{J} \quad \phi = \frac{TL}{JG} \quad \phi = \sum_i \frac{T_i L_i}{J_i G_i} \quad \phi = \int_0^L \frac{T}{JG} dx$$

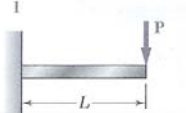
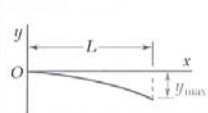
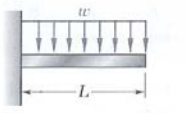
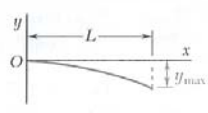


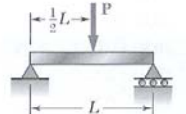
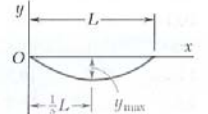
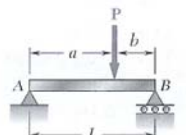
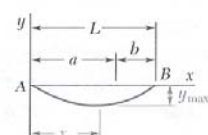
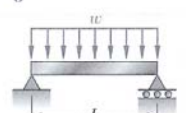
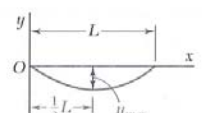

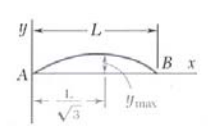
$$\frac{1}{\rho} = \frac{M}{EI} \quad \frac{1}{\rho'} = \frac{\nu}{\rho} \quad \sigma_x = -\frac{My}{I} \quad \sigma_x = \frac{P}{A} - \frac{My}{I}$$

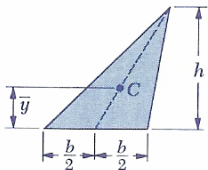
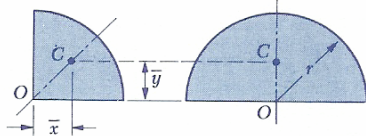
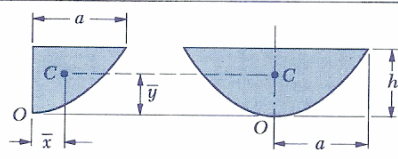
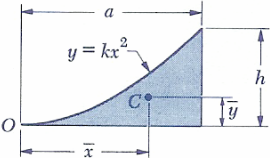
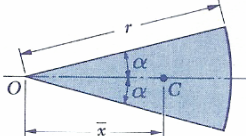
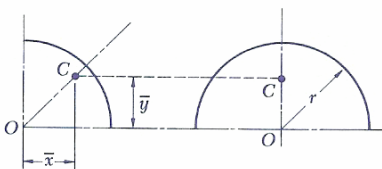
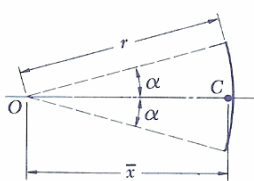
$$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} \quad \tan \phi = \frac{I_z}{I_y} \tan \theta$$

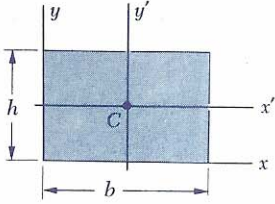
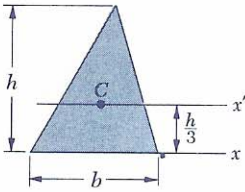
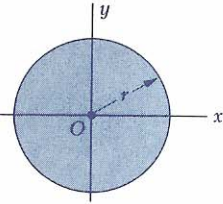
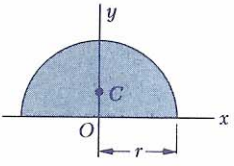
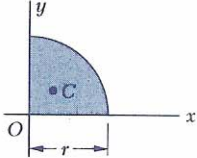
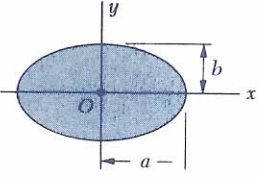
$$\frac{dV}{dx} = -w \quad V_D - V_c = -\int_{x_c}^{x_D} w dx$$

$$\frac{dM}{dx} = V \quad M_D - M_c = \int_{x_c}^{x_D} V dx$$

$$\frac{1}{\rho} = \frac{M(x)}{EI} \quad \frac{d^2 y}{dx^2} = \frac{M(x)}{EI}$$

Viga e carregamento	Linha elástica	Flèche máxima	Rotação na extremidade	Equação da linha elástica
		$-\frac{PL^3}{3EI}$	$-\frac{PL^2}{2EI}$	$y = \frac{P}{6EI}(x^3 - 3Lx^2)$
		$-\frac{pL^4}{8EI}$	$-\frac{pL^3}{6EI}$	$y = -\frac{P}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$
		$-\frac{ML^2}{2EI}$	$-\frac{ML}{EI}$	$y = -\frac{M}{2EI}x^2$
		$-\frac{PL^3}{48EI}$	$\pm \frac{PL^2}{16EI}$	Para $x \leq \frac{1}{2}L$: $y = \frac{P}{48EI}(4x^3 - 3L^2x)$
		Para $a > b$: $-\frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}EIL}$ em $x_m = \sqrt{\frac{L^2 - b^2}{3}}$	$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL}$ $\theta_B = +\frac{Pa(L^2 - a^2)}{6EIL}$	Para $x < a$: $y = \frac{Pb}{6EIL}[x^3 - (L^2 - b^2)x]$ Para $x = a$: $y = -\frac{Pa^2b^2}{3EIL}$
		$-\frac{5pL^4}{384EI}$	$\pm \frac{pL^3}{24EI}$	$y = -\frac{P}{24EI}(x^4 - 2Lx^3 + L^3x)$
		$\frac{ML^2}{9\sqrt{3}EI}$	$\theta_A = +\frac{ML}{6EI}$ $\theta_B = -\frac{ML}{3EI}$	$y = -\frac{M}{6EIL}(x^3 - L^2x)$

Shape		\bar{x}	\bar{y}	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$