PROBLEMA I — EXAME DE MEC. COMPUTACIONAL – ENG. BIOMÉDICA — 2ª ÉPOCA 21/07/2006 (1)

a)

$T = T_1 + T_2 + T_3$

$\phi = \frac{T_1 L_1}{J_1 G_1} = \frac{T_2 L_2}{J_2 G_2} = \frac{T_3 L_3}{J_3 G_3}$

$J_1 = \frac{1}{2}\pi r_1^4 = \frac{1}{2}\pi \times (6\times10^{-3})^4 = 2,036\times10^{-9}\,m^4$

$J_2 = \frac{1}{2}\pi (r_2^4 - r_1^4) = \frac{1}{2}\pi\left((9\times10^{-3})^4 - (6\times10^{-3})^4\right) = 8.27\times10^{-9}\,m^4$

$J_3 = \frac{1}{2}\pi (r_3^4 - r_2^4) = \frac{1}{2}\pi\left((15\times10^{-3})^4 - (9\times10^{-3})^4\right) = 6.92\times10^{-8}\,m^4$

$L_1 = L_2 = L_3$

$T_1 = \frac{J_1 G_1}{J_3 G_3} T_3 = 0.147\, T_3$

$T_2 = \frac{J_2 G_2}{J_3 G_3} T_3 = 0.018\, T_3$

$0.147\, T_3 + 0.018\, T_3 + T_3 = T \Rightarrow T_3 = 0.858\, T$ com $T = 500\,Nm$

$T_1 = 0.126\, T$

$T_2 = 0.155\, T$

$\tau_1 = \frac{T_1 r_1}{J_1} = 185.65\,MPa$ → Implante.

$\tau_2 = \frac{T_2 r_2}{J_2} = 8,43\,MPa$ → Aumento.

$\tau_3 = \frac{T_3 r_3}{J_3} = 93,99\,MPa$ → Osso.

b) Se G for todo igual.

$\tau_{máx} = \frac{T r_3}{J_{cheia}} = 94,34\,MPa$

$J_{cheia} = \frac{1}{2}\pi (15\times10^{-3})^4 = 7.95\times10^{-8}\,m^4$

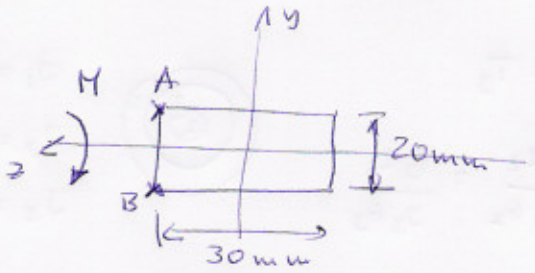
c) ϕ para a) $\phi = \frac{T_1 L_1}{J_1 G_1} = 0.093\,rad$

ϕ para b) $\phi = \frac{TL}{J_{cheia}\times G} = 0.094\,rad$

PROBLEMA 2

Secção em estudo.

a)



$e = 15\,mm - 10\,mm = 5\,mm$

$M = F \times e = 5000 \times 5 \times 10^{-3} = 25\,Nm$

NO PONTO B

$$\sigma_B = \frac{5000}{0.6\times10^{-3}} - \frac{25\times(10\times10^{-3})}{20\times10^{-9}}$$

$$= -4.16\,MPa$$

$$I = \frac{1}{12}(30\times10^{-3})\cdot(20\times10^{-3})^3$$

$$A = (30\times10^{-3})\times(20\times10^{-3})$$

NO PONTO A

$$\sigma_A = \frac{5000}{0.6\times10^{-3}} + \frac{25\times(10\times10^{-3})}{20\times10^{-9}} = 20.83\,MPa$$

b)

$$\sigma_A = \frac{15000}{30\times10^{-3}\times d} + \frac{\left[15000\times(15\times10^{-3}-d/2)\right]d/2}{\frac{1}{12}\times30\times10^{-3}\times d^3} \leq 55\times10^6$$

$$\Rightarrow$$

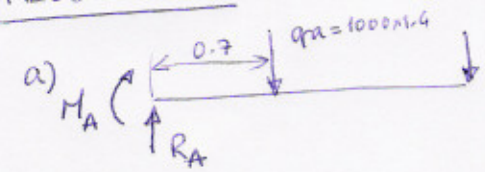
$$-55\times10^6 d^2 - 1\times10^6 d + 45\times10^3 \leq 0$$

$$\Rightarrow \quad d \geq 0.0209\,m$$

$$d \geq 20.9\,mm \approx 21\,mm$$

PROBLEMA 3

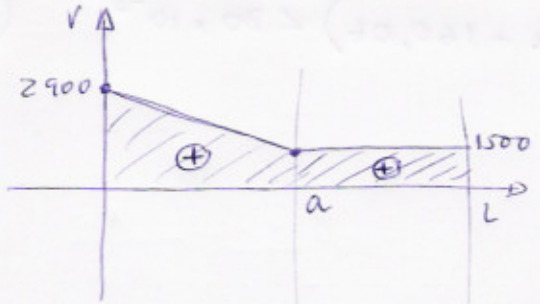
a)

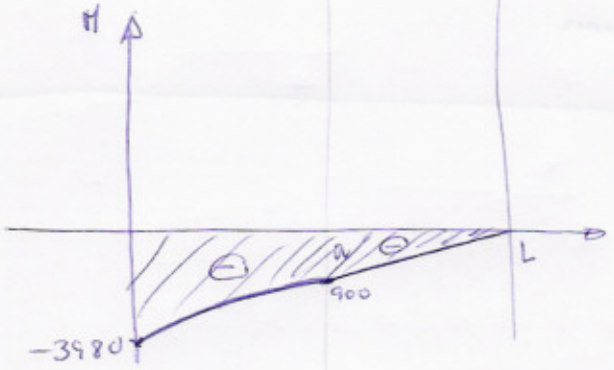


$\uparrow \sum F = 0 \quad R_A - 1400 - 1500 = 0 \Rightarrow R_A = 2900\,N$

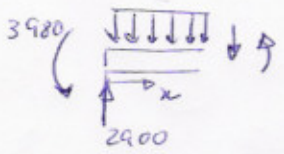
$\circlearrowleft \sum M_A = 0 \quad M_A + 1400\times0.7 + 1500\times2 = 0 \Rightarrow M_A = -3980\,Nm$

b)





entre 0 e a



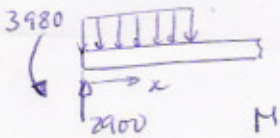
$V + 1000x - 2900 = 0$

$V = -1000x + 2900$

$M + 3980 - 2900x + 1000x \cdot \frac{x}{2}$

$M = -1000\frac{x^2}{2} + 2900x - 3980$

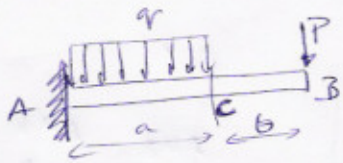
entre a e L



$M + 3980 - 2900x + 1400(x - 0.7) = 0$

$M = -3000 + 1500x$

c)



AC

$$EI\frac{dy_{AC}}{dx} = -1000\frac{x^3}{6} + 2900\frac{x^2}{2} - 3980x + C_1$$

$$EI\,y_{AC} = -1000\frac{x^4}{24} + 2900\frac{x^3}{6} - 3980\frac{x^2}{2} + C_1x + C_2$$

CB

$$EI\frac{dy}{dx} = -3000x + 1500\frac{x^2}{2} + C_3$$

$$EI\,y = -3000\frac{x^2}{2} + 1500\frac{x^3}{6} + C_3x + C_4$$

$x = 0 \quad \frac{dy}{dx} = 0 \Rightarrow C_1 = 0$

$x = 0 \quad y = 0 \Rightarrow C_2 = 0$

$x = a = 1.4 \Rightarrow \frac{dy_{AC}}{dx} = \frac{dy_{CB}}{dx}$ e $y_{AC} = y_{CB}$

$\Rightarrow \quad C_3 = -457,33$

$C_4 = 160,06$

$$y_{AC} = \left(-\frac{1000}{24}x^4 + \frac{2900}{6}x^3 - \frac{3980}{2}x^2\right) \times \frac{1}{EI}$$

$$y_{CB} = \left(-\frac{3000}{2}x^2 + \frac{1500}{6}x^3 - 457,33x + 160,06\right) \times \frac{1}{EI}$$

d) $y_{CB} = \frac{1}{EI}\left(-3000\frac{x^2}{2} + 1500\frac{x^3}{6} - 457{,}33x + 160{,}06\right) < 20\times10^{-3}$ (4)

para $x = 2$

$\Rightarrow \quad I \geq 3.396\times10^{-6}$

como $I = \frac{1}{4}\pi r^4 \Rightarrow \quad r \geq 45{,}6\text{ mm}$

$d \geq 91{,}2\text{ mm}$

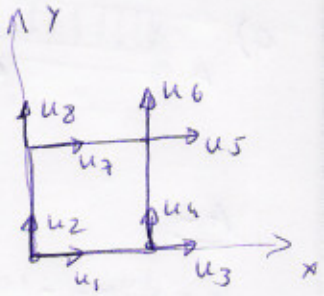
PROBLEMA 4

a) $\phi_1 = (x-1)(y-1)$ $\qquad \phi_3 = xy$

$\phi_2 = x(1-y)$ $\qquad \phi_4 = y(1-x)$

b) 4 nós K é uma matriz 8×8

$$\begin{bmatrix} 1/2 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 \\ & 1/2 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 \\ & & 1/2 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 \\ & & & 1/2 & 1/8 & 1/8 & 1/8 & 1/8 \\ & & & & 1/2 & 1/8 & 1/8 & 1/8 \\ & & & & & 1/2 & 1/8 & 1/8 \\ & & & & & & 1/2 & 1/8 \\ & & & & & & & 1/2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ P\cos30 \\ P\sin30 \\ 0 \\ 0 \end{Bmatrix}$$



como $u_1 = u_2 = u_3 = u_4 = u_7 = u_8 = 0$

o sistema a resolver fica

$$\begin{bmatrix} 1/2 & 1/8 \\ 1/8 & 1/2 \end{bmatrix} \begin{Bmatrix} u_5 \\ u_6 \end{Bmatrix} = \begin{Bmatrix} P\cos30 \\ P\sin30 \end{Bmatrix}$$

$\Rightarrow \quad u_5 = 1.5808\,P$

$u_6 = 0.6048\,P$

c) $\underset{\sim}{u}(x) = \begin{Bmatrix} u_x(x,y) \\ u_y(x,y) \end{Bmatrix} = \begin{Bmatrix} u_1\phi_1 + u_3\phi_2 + u_5\phi_3 + u_7\phi_4 \\ u_2\phi_1 + u_4\phi_2 + u_6\phi_3 + u_8\phi_4 \end{Bmatrix} = \begin{Bmatrix} 1.5808\,Pxy \\ 0.6048\,Pxy \end{Bmatrix}$

$\varepsilon_x = \frac{\partial u_x}{\partial x} = 1.5808\,yP$

$\varepsilon_y = \frac{\partial u_y}{\partial x} = 0.6048\,xP$

$\gamma_{xy} = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) = 1.5808\,xP + 0.6048\,yP$

no centro do elemento $x = 1/2$; $y = \frac{1}{2}$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \begin{Bmatrix} 0.7904\,P \\ 0.3024\,P \\ 1.0928\,P \end{Bmatrix}$$

$\sigma_x = 0.7904\,P$
$\sigma_y = 0.3024\,P$
$\tau_{xy} = 0.5464\,P$