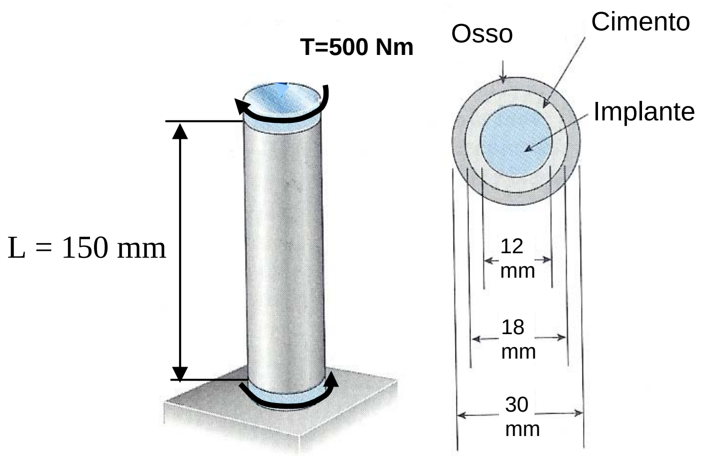
MECÂNICA COMPUTACIONAL
Licenciatura em Engenharia Biomédica

2ª Época Ano Lectivo de 2005/2006 21/Julho/2006

- O Exame é sem consulta. O formulário está anexo a este exame.
- Não são permitidos computadores pessoais ou telemóveis.
- Todas as folhas do exame deverão ser identificadas.
- A duração do exame é de 2 horas e 30 minutos.

Problema I (5 val.)

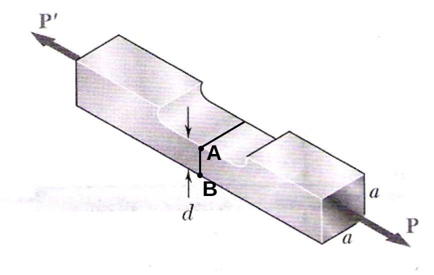
A figura representa um modelo simplificado para analisar um implante cimentado num osso longo. Sujeitou-se o conjunto a um momento torsor T=500 Nm aplicada como mostra a figura. Considere o módulo de rigidez transversal para o osso de 10 GPa, para o cimento de 1,5 GPa e para o Implante de 50 GPa.
a) Determine as tensões de corte máximas no implante, no cimento e no osso compacto.
b) Considere todos os materiais com rigidez ao corte igual à do osso (10 GPa). Nestas condições determine a tensão de corte máxima.
c) Determine o ângulo de torção para a situação da alínea a) e para a situação da alínea b)



Problema II (4 val.)

Retirou-se um porção de material a uma barra de secção transversal quadrada com a= 30 mm, obtendo-se o componente representado na figura. O carregamento P está alinhado com o eixo da peça original.

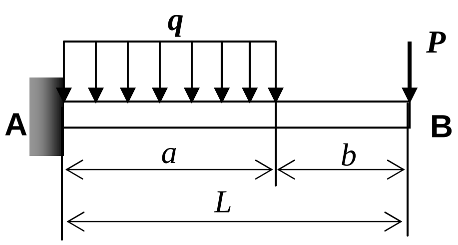
a) Para P = 5 KN e d =20 mm determine a tensão normal no pontos A e B.
b) Para P=15 KN e tensão normal admissível de 55 MPa, determine o valor mínimo da espessura d.



Problema III (6 val.)

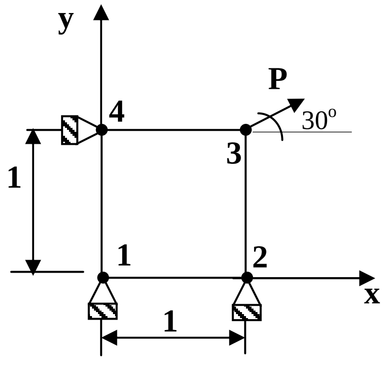
Considere o varão de alumínio (secção circular) cujo módulo de rigidez é E= 70 GPa, sujeito ao carregamento representado na figura. Considere L=2 metros, *a*=1.4 e *b*=0.6. O carregamento distribuído é de q=1000 N/m e a carga concentrada P=1500 N.

a) Calcule as reacções no apoio.
b) Desenhe os diagramas de esforço transverso e momento flector, e indique os respectivos valores máximos absolutos.
c) Calcule a equação da curva elástica da viga (em função de EI).
d) Sabendo que o deslocamento vertical em B não pode exceder 20 mm, determine o diâmetro mínimo que o varão pode ter.



Problema IV (5 val.)

Considere o elemento plano de quatro nós, apresentado na figura, para um problema de deformação plana. Admita E=1 e ν=0.



(a) Estabeleça as funções de base para este elemento (no referencial x-y).
(b) Admita que os elementos da diagonal da matriz de rigidez valem 1/2 e os elementos fora da diagonal 1/8, determine os deslocamentos nodais para as condições indicadas na figura.
(c) Determine a tensão no centro do elemento.

FORMULÁRIO:

$$\sigma = \frac{P}{A}$$ $$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$ $$\tau = \lim_{\Delta A \to 0} \frac{\Delta V}{\Delta A}$$ $$\tau_{xy} = \tau_{yx}$$ $$\sigma = \frac{P}{A_0}\cos^2\theta$$

$$\tau = \frac{P}{A_0}\cos\theta\sin\theta$$ $$\varepsilon = \frac{d\delta}{dx}$$ $$\sigma = E\varepsilon$$ $$\varepsilon_T = \alpha\Delta T$$ $$\delta = \frac{PL}{AE}$$ $$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

$$\delta = \int_0^L \frac{P}{AE}dx$$ $$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$ $$\varepsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$ $$\varepsilon_z = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$\tau_{xy} = G\gamma_{xy}$$ $$\tau_{yz} = G\gamma_{yz}$$ $$\tau_{xz} = G\gamma_{xz}$$ $$G = \frac{E}{2(1+\nu)}$$ $$\gamma = \frac{\rho\phi}{L}$$ $$\tau = \frac{T\rho}{J}$$

$$\phi = \frac{TL}{JG}$$ $$\phi = \sum_i \frac{T_i L_i}{J_i G_i}$$ $$\phi = \int_0^L \frac{T}{JG}dx$$ $$\frac{1}{\rho} = \frac{M}{EI}$$ $$\frac{1}{\rho'} = -\frac{\nu}{\rho}$$ $$\sigma_x = -\frac{My}{I}$$

$$\sigma_x = \frac{P}{A} - \frac{My}{I}$$ $$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$ $$\tan\phi = \frac{I_z}{I_y}\tan\theta$$ $$\frac{dV}{dx} = -w$$

$$V_D - V_c = -\int_{x_C}^{x_D} w\,dx$$ $$\frac{dM}{dx} = V$$ $$M_D - M_c = \int_{x_C}^{x_D} V\,dx$$ $$\frac{1}{\rho} = \frac{M(x)}{EI}$$ $$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

$$K_{ij}^e = \int_0^{h^e} EA\frac{d\phi_i}{dx}\frac{d\phi_j}{dx}dx$$ $$F_i^e = \int_0^{h^e} f\phi_i dx$$ $$\left[K_{ij}^e\right] = \frac{E^e A^e}{L^e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$ $$\left\{F_i^e\right\} = \frac{f^e L^e}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$\phi_1 = 1 - \frac{x}{h^e}$$ $$\phi_2 = \frac{x}{h^e}$$ $$K_{ij}^e = \int_0^{h^e} EI\frac{d^2\phi_i}{dx^2}\frac{d^2\phi_j}{dx^2}dx$$ $$F_i^e = \int_0^{h^e} q\phi_i dx$$

$$\phi_1 = 1 - 3\left(\frac{x}{h^e}\right)^2 + 2\left(\frac{x}{h^e}\right)^3$$ $$\phi_2 = -x\left(1 - \frac{x}{h^e}\right)^2$$ $$\phi_3 = 3\left(\frac{x}{h^e}\right)^2 - 2\left(\frac{x}{h^e}\right)^3$$ $$\phi_4 = -x\left[\left(\frac{x}{h^e}\right)^2 - \frac{x}{h^e}\right]$$

$$\left[K_{ij}^e\right] = \frac{2EI}{h_e^3}\begin{bmatrix} 6 & -3h_e & -6 & -3h_e \\ -3h_e & 2h_e^2 & 3h_e & h_e^2 \\ -6 & 3h_e & 6 & 3h_e \\ -3h_e & h_e^2 & 3h_e & 2h_e^2 \end{bmatrix}$$ $$\left\{F_i^e\right\} = \frac{qh_e}{12}\begin{Bmatrix} 6 \\ -h_e \\ 6 \\ h_e \end{Bmatrix}$$ $$K_{\alpha\beta}^e = \int_{\Omega^e} k_{ij}\frac{\partial\phi_\alpha}{\partial x_i}\frac{\partial\phi_\beta}{\partial x_j}d\Omega$$ $$F_\alpha^e = \int_{\Gamma_e} q\phi_\alpha d\Gamma$$

$$\phi_1 = \frac{1}{4}(1-\xi)(1-\eta)$$ $$\phi_2 = \frac{1}{4}(1+\xi)(1-\eta)$$ $$\phi_3 = \frac{1}{4}(1+\xi)(1+\eta)$$ $$\phi_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

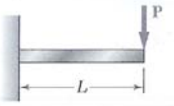
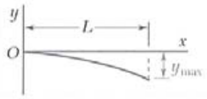
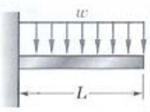
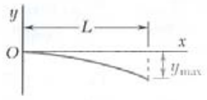
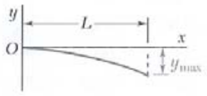
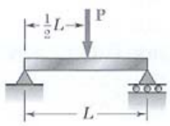
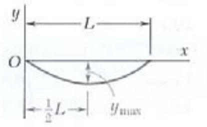
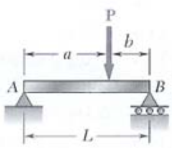
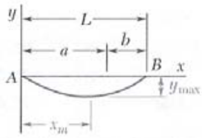
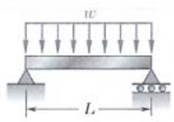
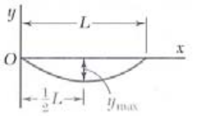
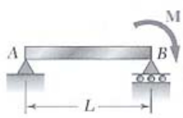
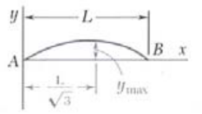
deformação plana

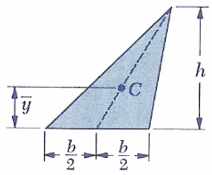
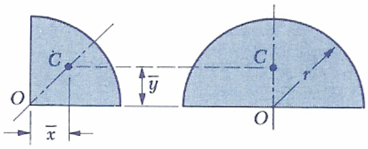
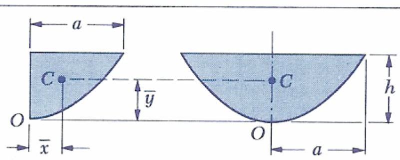
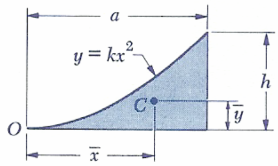
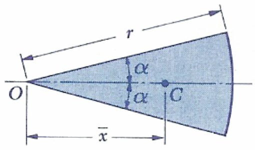
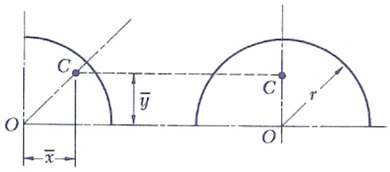
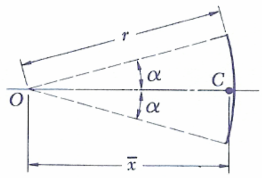
$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\upsilon)(1-2\upsilon)}\begin{bmatrix} 1-\upsilon & \upsilon & 0 \\ \upsilon & 1-\upsilon & 0 \\ 0 & 0 & \frac{1-2\upsilon}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

tensão plana

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1-\upsilon^2)}\begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \frac{1-\upsilon}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$ $$\varepsilon_{ij} = \frac{1}{2}\gamma_{ij}$$

Viga e carregamento	Linha elástica	Flecha máxima	Rotação na extremidade	Equação da linha elástica
1 		$-\frac{PL^3}{3EI}$	$-\frac{PL^2}{2EI}$	$y=\frac{P}{6EI}(x^3-3Lx^2)$
2 		$-\frac{pL^4}{8EI}$	$-\frac{pL^3}{6EI}$	$y=-\frac{p}{24EI}(x^4-4Lx^3+6L^2x^2)$
3 		$-\frac{ML^2}{2EI}$	$-\frac{ML}{EI}$	$y=-\frac{M}{2EI}x^2$
4 		$-\frac{PL^3}{48EI}$	$\pm\frac{PL^2}{16EI}$	Para $x\le\frac{1}{2}L$: $y=\frac{P}{48EI}(4x^3-3L^2x)$
5 		Para $a>b$: $-\frac{Pb(L^2-b^2)^{3/2}}{9\sqrt{3}EIL}$ em $x_m=\sqrt{\frac{L^2-b^2}{3}}$	$\theta_A=-\frac{Pb(L^2-b^2)}{6EIL}$ $\theta_B=+\frac{Pa(L^2-a^2)}{6EIL}$	Para $x<a$: $y=\frac{Pb}{6EIL}[x^3-(L^2-b^2)x]$ Para $x=a$: $y=-\frac{Pa^2b^2}{3EIL}$
6 		$-\frac{5pL^4}{384EI}$	$\pm\frac{pL^3}{24EI}$	$y=-\frac{p}{24EI}(x^4-2Lx^3+L^3x)$
7 		$\frac{ML^2}{9\sqrt{3}EI}$	$\theta_A=+\frac{ML}{6EI}$ $\theta_B=-\frac{ML}{3EI}$	$y=-\frac{M}{6EIL}(x^3-L^2x)$

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

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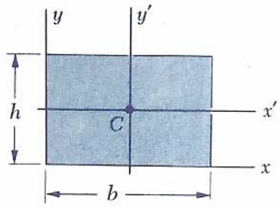
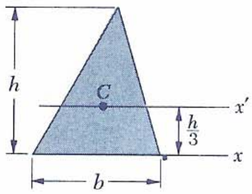
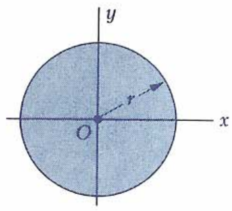
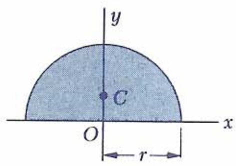
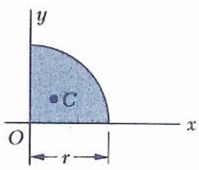
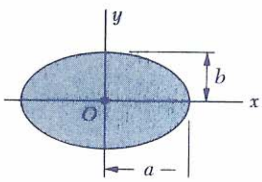
<tr><td>Rectangle</td><td>
</td><td>$\bar{I}_{x'} = \frac{1}{12}bh^3$
$\bar{I}_{y'} = \frac{1}{12}b^3h$
$I_x = \frac{1}{3}bh^3$
$I_y = \frac{1}{3}b^3h$
$J_C = \frac{1}{12}bh(b^2 + h^2)$</td></tr>
<tr><td>Triangle</td><td>
</td><td>$\bar{I}_{x'} = \frac{1}{36}bh^3$
$I_x = \frac{1}{12}bh^3$</td></tr>
<tr><td>Circle</td><td>
</td><td>$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$
$J_O = \frac{1}{2}\pi r^4$</td></tr>
<tr><td>Semicircle</td><td>
</td><td>$I_x = I_y = \frac{1}{8}\pi r^4$
$J_O = \frac{1}{4}\pi r^4$</td></tr>
<tr><td>Quarter circle</td><td>
</td><td>$I_x = I_y = \frac{1}{16}\pi r^4$
$J_O = \frac{1}{8}\pi r^4$</td></tr>
<tr><td>Ellipse</td><td>
</td><td>$\bar{I}_x = \frac{1}{4}\pi ab^3$
$\bar{I}_y = \frac{1}{4}\pi a^3b$
$J_O = \frac{1}{4}\pi ab(a^2 + b^2)$</td></tr>
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