

MECÂNICA E MODELAÇÃO COMPUTACIONAL
Mestrado Integrado em Engenharia Biomédica

1ª Época

Ano Lectivo de 2012/2013

4/Janeiro/2013

- O Exame é sem consulta. O formulário está anexo a este exame.
- Não são permitidos computadores pessoais ou telemóveis.
- Todas as folhas do exame deverão ser identificadas.
- A duração do exame é de 2 horas e 30 minutos.

Problema I (4 val.)

As barras AB e DC feitas em alumínio ($E=75\text{Ga}$) e com área da secção transversal de 125 mm^2 , suportam a barra rígida BC. Considerando uma carga aplicada de 5 kN como mostra a figura e desprezando o peso próprio da barra rígida e das barras, determine:

- a) A tensão normal em cada uma das barras AB e CD. (2 val.)
- b) O deslocamento vertical do ponto E. (2 val.)

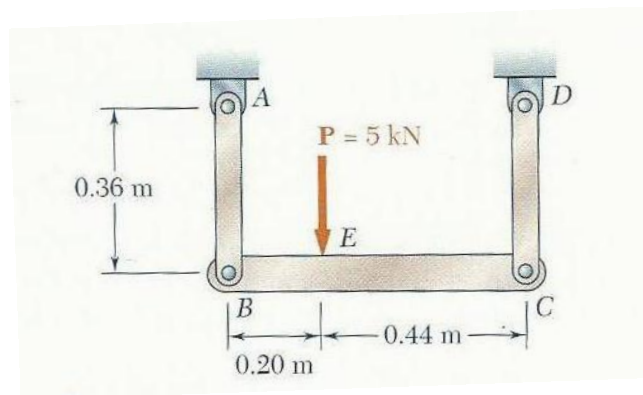


Figura 1

Problema II (5 val.)

Considere o diagrama da figura 2, o qual pretende representar a carga a que o fémur está sujeito devido a uma queda. O valor da força F é de 2800 N , $\theta=30^\circ$ e $L=25\text{ mm}$. Admite-se que o colo do fémur (secção A-A) é só constituído por uma coroa de osso cortical de diâmetro exterior $D_{\text{ext}}=28\text{ mm}$ e $d_{\text{int}}=23\text{ mm}$. Nestas condições determine:

- a) O momento resultante na secção A-A (1 val.)
- b) A tensão normal nos pontos s, e i. (2 val.)
- c) Uma das consequências da osteoporose é a diminuição da espessura de osso cortical. Admitindo que essa diminuição se dá nesta região pelo aumento do diâmetro interior, mantendo-se o diâmetro exterior, e que a tensão normal admissível do osso é de $\sigma_{\text{adm}}=60\text{MPa}$ determine o diâmetro interior máximo admissível. Considere o mesmo valor de tensão admissível para tracção e para compressão. (2 val.)

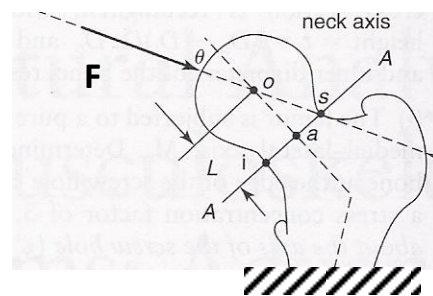
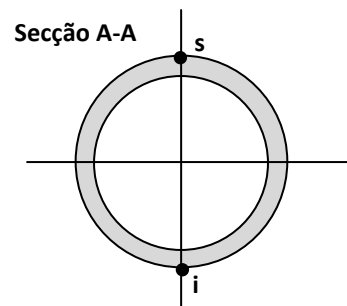


Figura 2



Problema III (5 val.)

Considere a viga representada na figura 3. A viga está fixa numa extremidade e apoiado na outra extremidade. O material da viga é alumínio com $E=70GPa$. O comprimento entre apoios é $L=2$ metros e a secção da viga é rectangular com $I=5 \times 10^{-6} m^4$. O carregamento distribuído é de $q=1000 N/m$.

- Determine as reacções nos apoios (2 val.)
- Determine a equação da curva elástica (2 val.)
- Determine a deformada a meio vão (ponto C) (1 val.)

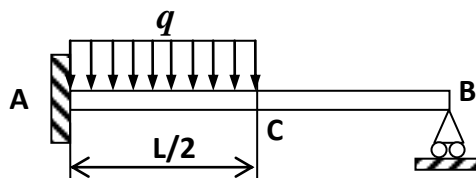


Figura 3

Problema IV (6 val.)

Pretende-se resolver o problema III utilizando dois elementos de viga de igual comprimento.

- Determine a matriz global do sistema (antes das condições de fronteira). (1,5 val.)
- Determine o vector de cargas global. (1,5 val.)
- Imponha as condições de fronteira e determine o deslocamento vertical no ponto C. (2 val.)
- Determine o valor do momento flector para $x=L/4$. (1 val.)

FORMULÁRIO:

$$\sigma = \frac{P}{A} \quad \sigma = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A} \quad \tau = \lim_{\Delta A \rightarrow 0} \frac{\Delta V}{\Delta A} \quad \tau_{xy} = \tau_{yx} \quad \sigma = \frac{P}{A_0} \cos^2 \theta$$

$$\tau = \frac{P}{A_0} \cos \theta \sin \theta \quad \varepsilon = \frac{d\delta}{dx} \quad \sigma = E\varepsilon \quad \varepsilon_T = \alpha \Delta T \quad \delta = \frac{PL}{AE}$$

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i} \quad \delta = \int_0^L \frac{P}{AE} dx$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E} \quad \varepsilon_y = -\nu \frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E} \quad \varepsilon_z = -\nu \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$\tau_{xy} = G\gamma_{xy} \quad \tau_{yz} = G\gamma_{yz} \quad \tau_{xz} = G\gamma_{xz} \quad G = \frac{E}{2(1+\nu)}$$

$$\gamma = \frac{\rho\phi}{L} \quad \tau = \frac{T\rho}{J} \quad \phi = \frac{TL}{JG} \quad \phi = \sum_i \frac{T_i L_i}{J_i G_i} \quad \phi = \int_0^L \frac{T}{JG} dx$$

$$\frac{1}{\rho} = \frac{M}{EI} \quad \frac{1}{\rho'} = \frac{\nu}{\rho} \quad \sigma_x = -\frac{My}{I} \quad \sigma_x = \frac{P}{A} - \frac{My}{I}$$

$$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} \quad \tan \phi = \frac{I_z}{I_y} \tan \theta \quad \frac{dV}{dx} = -w \quad V_D - V_C = -\int_{x_C}^{x_D} w dx$$

$$\frac{dM}{dx} = V \quad M_D - M_C = \int_{x_C}^{x_D} V dx \quad \frac{1}{\rho} = \frac{M(x)}{EI} \quad \frac{d^2 y}{dx^2} = \frac{M(x)}{EI}$$

$$K_{ij}^e = \int_0^{h^e} EA \frac{d\phi_i}{dx} \frac{d\phi_j}{dx} dx \quad F_i^e = \int_0^{h^e} f \phi_i dx \quad [K_{ij}^e] = \frac{E^e A^e}{L^e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad \{F_i^e\} = \frac{f^e L^e}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\phi_1 = 1 - \frac{x}{h^e} \quad \phi_2 = \frac{x}{h^e} \quad K_{ij}^e = \int_0^{h^e} EI \frac{d^2 \phi_i}{dx^2} \frac{d^2 \phi_j}{dx^2} dx \quad F_i^e = \int_0^{h^e} q \phi_i dx$$

$$\phi_1 = 1 - 3\left(\frac{\bar{x}}{h^e}\right)^2 + 2\left(\frac{\bar{x}}{h^e}\right)^3 \quad \phi_2 = -\bar{x}\left(1 - \frac{\bar{x}}{h^e}\right) \quad \phi_3 = 3\left(\frac{\bar{x}}{h^e}\right)^2 - 2\left(\frac{\bar{x}}{h^e}\right)^3 \quad \phi_4 = -\bar{x}\left[\left(\frac{\bar{x}}{h^e}\right)^2 - \frac{\bar{x}}{h^e}\right]$$

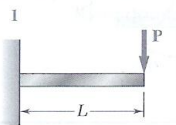
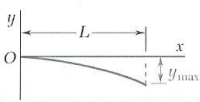
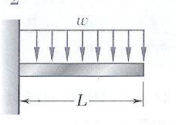
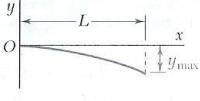
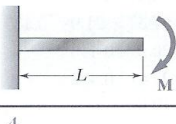
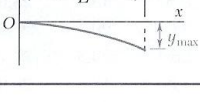
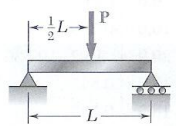
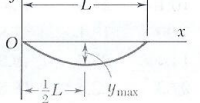
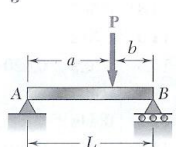
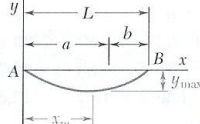
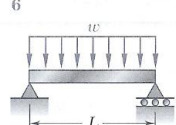
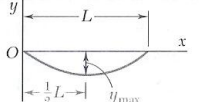
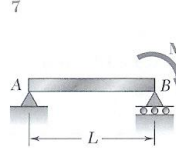
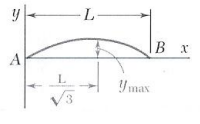
$$[K_{ij}^e] = \frac{2EI}{h_e^3} \begin{bmatrix} 6 & -3h_e & -6 & -3h_e \\ -3h_e & 2h_e^2 & 3h_e & h_e^2 \\ -6 & 3h_e & 6 & 3h_e \\ -3h_e & h_e^2 & 3h_e & 2h_e^2 \end{bmatrix} \quad \{F_i^e\} = \frac{qh_e}{12} \begin{Bmatrix} 6 \\ -h_e \\ 6 \\ h_e \end{Bmatrix} \quad K_{\alpha\beta}^e = \int_{\Omega^e} k_{ij} \frac{\partial \phi_\alpha}{\partial x_i} \frac{\partial \phi_\beta}{\partial x_j} d\Omega \quad F_\alpha^e = \int_{\Gamma_e} q \phi_\alpha d\Gamma$$

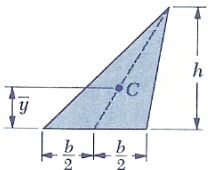
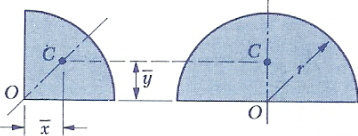
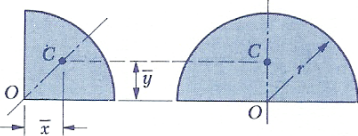
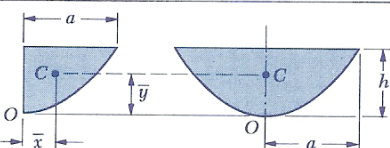
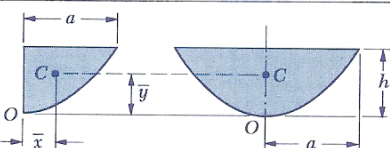
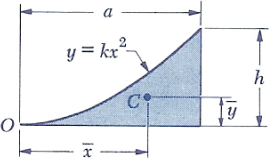
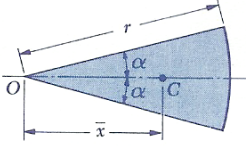
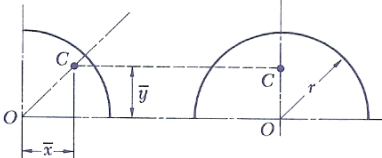
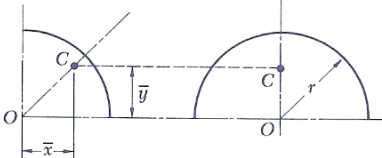
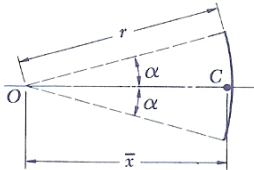
$$\phi_1 = \frac{1}{4}(1-\xi)(1-\eta) \quad \phi_2 = \frac{1}{4}(1+\xi)(1-\eta) \quad \phi_3 = \frac{1}{4}(1+\xi)(1+\eta) \quad \phi_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

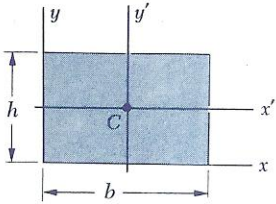
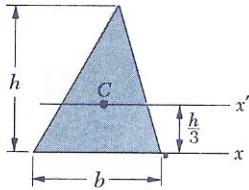
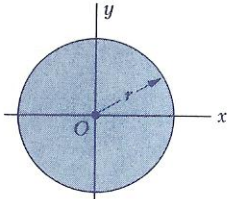
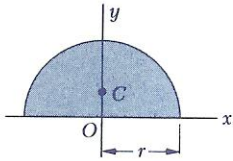
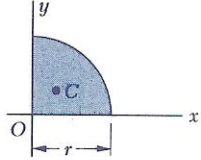
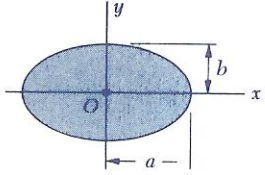
$$\int_{-1}^1 \int_{-1}^1 F(\xi, \eta) d\xi d\eta = \sum_{I=1}^M \sum_{J=1}^N F(\xi_I, \eta_J) W_I W_J \quad \int_{-1}^1 F(\xi) d\xi = \sum_{i=1}^N F(\xi_i) W_i$$

$$K^e = \frac{2EI}{h^3} \begin{bmatrix} \mu c^2 + 6s^2 & (\mu-6)cs & 3hs & -(\mu c^2 + 6s^2) & -(\mu-6)cs & 3hs \\ & \mu s^2 + 6c^2 & -3hc & -(\mu-6)cs & -(\mu s^2 + 6c^2) & -3hc \\ & & 2h^2 & -3hs & 3hc & h^2 \\ \dots & \dots & \dots & \mu c^2 + 6s^2 & (\mu-6)cs & -3hs \\ & & & & \mu s^2 + 6c^2 & 3hc \\ & & & & & 2h^2 \end{bmatrix}$$

$$\mu = \frac{Ah^2}{2I}, \quad c = \cos(\alpha), \quad s = \sin(\alpha)$$

Viga e carregamento	Linha elástica	Flèche máxima	Rotação na extremidade	Equação da linha elástica
1 		$-\frac{PL^3}{3EI}$	$-\frac{PL^2}{2EI}$	$y = \frac{P}{6EI}(x^3 - 3Lx^2)$
2 		$-\frac{pL^4}{8EI}$	$-\frac{pL^3}{6EI}$	$y = -\frac{p}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$
3 		$-\frac{ML^2}{2EI}$	$-\frac{ML}{EI}$	$y = -\frac{M}{2EI}x^2$
4 		$-\frac{PL^3}{48EI}$	$\pm \frac{PL^2}{16EI}$	Para $x \leq \frac{1}{2}L$: $y = \frac{P}{48EI}(4x^3 - 3L^2x)$
5 		Para $a > b$: $-\frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}EIL}$ em $x_m = \sqrt{\frac{L^2 - b^2}{3}}$	$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL}$ $\theta_B = +\frac{Pa(L^2 - a^2)}{6EIL}$	Para $x < a$: $y = \frac{Pb}{6EIL}[x^3 - (L^2 - b^2)x]$ Para $x = a$: $y = -\frac{Pa^2b^2}{3EIL}$
6 		$-\frac{5pL^4}{384EI}$	$\pm \frac{pL^3}{24EI}$	$y = -\frac{p}{24EI}(x^4 - 2Lx^3 + L^3x)$
7 		$\frac{ML^2}{9\sqrt{3}EI}$	$\theta_A = +\frac{ML}{6EI}$ $\theta_B = -\frac{ML}{3EI}$	$y = -\frac{M}{6EIL}(x^3 - L^2x)$

Shape		\bar{x}	\bar{y}	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

<p>Rectangle</p>		$\begin{aligned} \bar{I}_{x'} &= \frac{1}{12}bh^3 \\ \bar{I}_{y'} &= \frac{1}{12}b^3h \\ I_x &= \frac{1}{3}bh^3 \\ I_y &= \frac{1}{3}b^3h \\ J_C &= \frac{1}{12}bh(b^2 + h^2) \end{aligned}$
<p>Triangle</p>		$\begin{aligned} \bar{I}_x &= \frac{1}{36}bh^3 \\ I_x &= \frac{1}{12}bh^3 \end{aligned}$
<p>Circle</p>		$\begin{aligned} \bar{I}_x &= \bar{I}_y = \frac{1}{4}\pi r^4 \\ J_O &= \frac{1}{2}\pi r^4 \end{aligned}$
<p>Semicircle</p>		$\begin{aligned} I_x &= I_y = \frac{1}{8}\pi r^4 \\ J_O &= \frac{1}{4}\pi r^4 \end{aligned}$
<p>Quarter circle</p>		$\begin{aligned} I_x &= I_y = \frac{1}{16}\pi r^4 \\ J_O &= \frac{1}{8}\pi r^4 \end{aligned}$
<p>Ellipse</p>		$\begin{aligned} \bar{I}_x &= \frac{1}{4}\pi ab^3 \\ \bar{I}_y &= \frac{1}{4}\pi a^3b \\ J_O &= \frac{1}{4}\pi ab(a^2 + b^2) \end{aligned}$