PROBLEMA I

MMC

1º ÉPOCA

15/01/2011

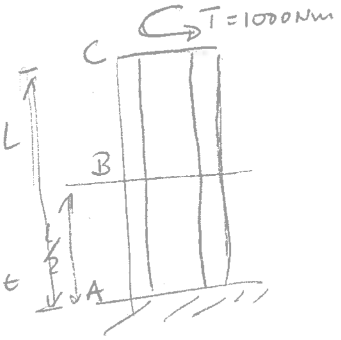
(1)

a) NA SECÇÃO B-B $T = 1000\,Nm$

e $\tau = \dfrac{TC}{J}$ com $J = \dfrac{1}{2}\pi \times ((15\times10^{-3})^4 - (9\times10^{-3})^4) = 6,92156\times10^{-8}$

e $C = 15\times10^{-3}$

$$\tau = \frac{1000\times15\times10^{-3}}{6.92156\times10^{-8}} = 216,71\times10^{6}\,Pa = 216,71\,MPa$$



Na secção A-A

$T = 1000\,Nm = T_{osso} + T_{haste}$

mas sabemos que $\phi_{osso} = \phi_{haste}$ PARA A PARTE ONDE EXISTE HASTE (ATÉ L/2)

Logo:

$$\frac{T_{osso}\times L/2}{J_{osso}\times G_{osso}} = \frac{T_{HASTE}\times L/2}{J_{HASTE}\times G_{HASTE}}$$

$$\frac{T_{osso}}{J_{osso}G_{osso}} = \frac{(T - T_{osso})}{J_{HASTE}G_{HASTE}}$$

$$T_{osso}(J_{HASTE}G_{HASTE} + T_{osso}G_{osso}) = TJ_{osso}G_{osso}$$

$$T_{osso} = \frac{J_{osso}G_{osso}}{(J_{HASTE}G_{HASTE} + J_{osso}G_{osso})}\times T$$

$$T_{osso} = \frac{6,92156\times10^{-8}\times10\times10^{9}}{(1.0306\times10^{-8}\times55\times10^{9} + 6.92156\times10^{-8}\times10\times10^{9})}\times1000 =$$

$T_{osso} = 549,77\,Nm$

$J_{HASTE} = \frac{1}{2}\pi\times(9\times10^{-3})^4 = 1,0306\times10^{-8}$

$T_{HASTE} = 450,22\,Nm$

$$\tau_{osso} = \frac{549,77\times15\times10^{-3}}{6.92156\times10^{-8}} = 119,14\times10^{6}\,Pa = 119,14\,MPa$$

b) $$\tau_{HASTE} = \frac{450,22\times9\times10^{-3}}{1.0306\times10^{-8}} = 393,17\times10^{6}\,Pa = 393,17\,MPa.$$

c) $$\phi_C = \phi_{C/B} + \phi_{B/A} = \frac{(T_{osso})_{B-B}\times\frac{L}{2}}{(J_{osso})(G_{osso})} + \frac{(T_{osso})_{A-A}\times\frac{L}{2}}{(J_{osso})(G_{osso})} =$$

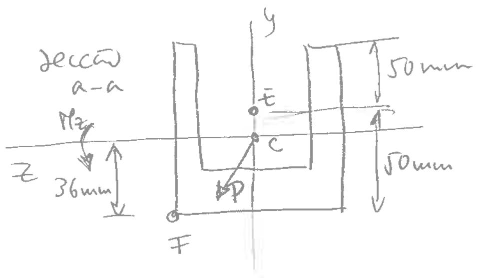
$$= \frac{1000\times50\times10^{-3} + 549,77\times50\times10^{-3}}{6.92156\times10^{-8}\times10\times10^{9}} = 0.112\,rad =$$

$= \underline{6,41^{\circ}}$

d) A tensão normal máxima num componente sujeito a torção ocorre num plano a 45° e vale

$$\sigma_{máx} = \frac{Tc}{J} = 216,71\ MPa$$

PROBLEMA II



Devido à carga de tracção aplicada no eixo que passa por E nesta secção as "forças" internas são:

$$P = 60\ kN \quad (\text{tracção})$$

$$M_z = -60000 \times (50\times10^{-3} - 36\times10^{-3})\ Nm$$ (momento em torno do z negativo)

$$M_z = -840\ Nm$$

a) A tensão normal máxima só pode ocorrer para $y = 64mm$ ou $y = -36\ mm$ (ponto F)

e $$\sigma = \frac{P}{A} - \frac{M_z\, y}{I_z}$$

Como M_z é negativo P positivo, a tensão máxima será para $y = 64\ mm$.

$$A = 2\times\left[(100\times10^{-3})\times(6\times10^{-3})\right] + \left[(88\times10^{-3})\times(6\times10^{-3})\right] = 1.728\times10^{-3}\ m$$

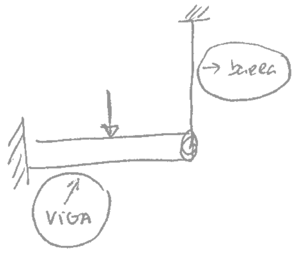
$$\sigma = \frac{60\times10^3}{1,728\times10^{-3}} - \frac{(-840)\times(64\times10^{-3})}{2\times10^{-6}}$$

$$= 61,60\times10^6\ Pa = \underline{\underline{61,60\ MPa}}$$

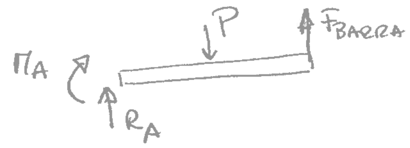
b) $$\sigma_F = \frac{60\times10^3}{1,728\times10^{-3}} - \frac{(-840)(-36\times10^{-3})}{2\times10^{-6}} = -19,60\times10^6\ Pa = \underline{\underline{-19,60\ MPa}}$$

PROBLEMA III

a)



EQUILÍBRIO DE CORPO LIVRE PARA A VIGA



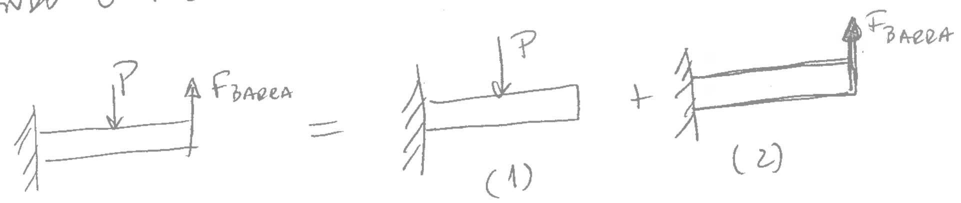
$\circlearrowleft + \sum M_A = 0 \qquad -M_A - P\times\frac{L}{2} + F_{BARRA}\times L = 0$

$+\uparrow \sum F = 0 \qquad R_A - P + F_{BARRA} = 0$

2 eqs. 3 INCOG. (M_A, R_A, F_{BARRA})

O PROBLEMA É ESTATICAMENTE INDETERMINADO.

UTILIZANDO O MÉTODO DA SOBREPOSIÇÃO:

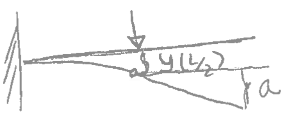


A CONDIÇÃO DA DEFORMADA DA VIGA É:

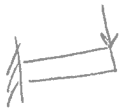
$$y_B^{(1)} + y_B^{(2)} = -\underbrace{\frac{F_{BARRA}\, L_{BARRA}}{E_{BARRA}\, A_{BARRA}}}_{\delta_{BARRA}}$$

(COM SINAL NEGATIVO PORQUE É NO SENTIDO NEGATIVO DA DEFORMADA DA VIGA)

$y_B^{(1)}$ e $y_B^{(2)}$ OBTÊM-SE DAS TABELAS

(1) 

$a = \theta(\frac{L}{2})\times(\frac{L}{2})$

$y(\frac{L}{2})$ É A FLECHA MÁXIMA PARA O CASO  COM $L = \frac{L}{2}$

LOGO $= -\frac{P(\frac{L}{2})^3}{3EI} = -\frac{(2\times10^3)\times 1^3}{3\times70\times10^9\times5\times10^{-6}} = -1.905\times10^{-3}$

$\theta(\frac{L}{2})$ É A ROTAÇÃO NA EXTREMIDADE PARA O MESMO CASO

$-\frac{P(L/2)^2}{2EI} = -\frac{2\times10^3\times1}{2\times70\times10^9\times5\times10^{-6}} = -2,857\times10^{-3}$ rad

LOGO $y_B^{(1)} = -1.905\times10^{-3} - (2.857\times10^{-3})\times 1 = -4,762\times10^{-3}$

(2) $y_B^{(2)}$ É A FLECHA MÁXIMA PARA O CASO 

COM A FORÇA NO SENTIDO CONTRÁRIO LOGO:

$$y_B^{(2)} = + \frac{F_{BARRA} \times L^3}{3EI} = + \frac{F_{BARRA} \times 2^3}{3 \times 70 \times 10^9 \times 5 \times 10^{-6}} = 7{,}619 \times 10^{-6}\, F_{BARRA}$$

LOGO DE $y_B^{(1)} + y_B^{(2)} = - \dfrac{F_{BARRA}\, L_{BARRA}}{E_{BARRA}\, A_{BARRA}}$

$$-4{,}762 \times 10^{-3} + 7{,}619 \times 10^{-6}\, F_{BARRA} = - F_{BARRA} \times \frac{2}{70 \times 10^9 \times 1 \times 10^{-3}}$$

$$7{,}619 \times 10^{-6}\, F_{BARRA} + 9{,}524 \times 10^{-9}\, F_{BARRA} = 4{,}762 \times 10^{-3}$$

$$7{,}629 \times 10^{-6}\, F_{BARRA} = 4{,}762 \times 10^{-3}$$

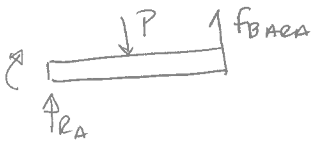
$$F_{BARRA} = 0{,}624 \times 10^3\, N = \underline{\underline{0{,}624\, kN}}$$

A FORÇA DE TRACÇÃO NA BARRE É $\underline{0{,}624\, kN}$

b) $\delta_B = \delta_{BARRA} = - \dfrac{624 \times 2}{70 \times 10^9 \times 1 \times 10^{-3}} = -17{,}828 \times 10^{-6}\, m = -17{,}83\, \mu m$

c) Reacções em A

DIAGRAMA CORPO LIVRE PARA A VIGA:



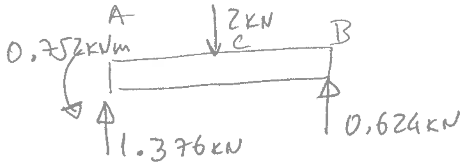
$$\begin{cases} -M_A - P \times \dfrac{L}{2} + F_{BARRA} \times L = 0 & \circlearrowleft \Sigma M_A = 0 \\ R_A - P + F_{BARRA} = 0 & \uparrow \Sigma F = 0 \end{cases}$$

$R_A - 2 + 0{,}624 = 0 \Rightarrow \underline{\underline{R_A = 1{,}376\, kN}}$

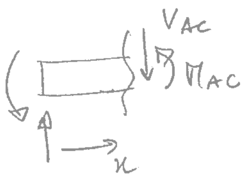
$-M_A - 2 \times 1 + 0{,}624 \times 2 = 0 \Rightarrow$

$M_A = 2 \times 0{,}624 - 2 = \underline{\underline{-0{,}752\, kNm}}$

II - a)



AC



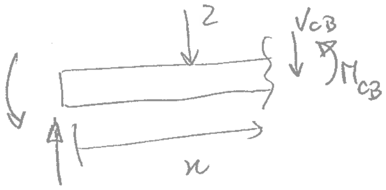
$V_{AC} - 1{,}376 = 0$

$V_{AC} = 1{,}376 \text{ kN}$

$M_{AC} + 0{,}752 - 1{,}376\,x = 0$

$M_{AC} = 1{,}376x - 0{,}752 \text{ kNm}$

CB



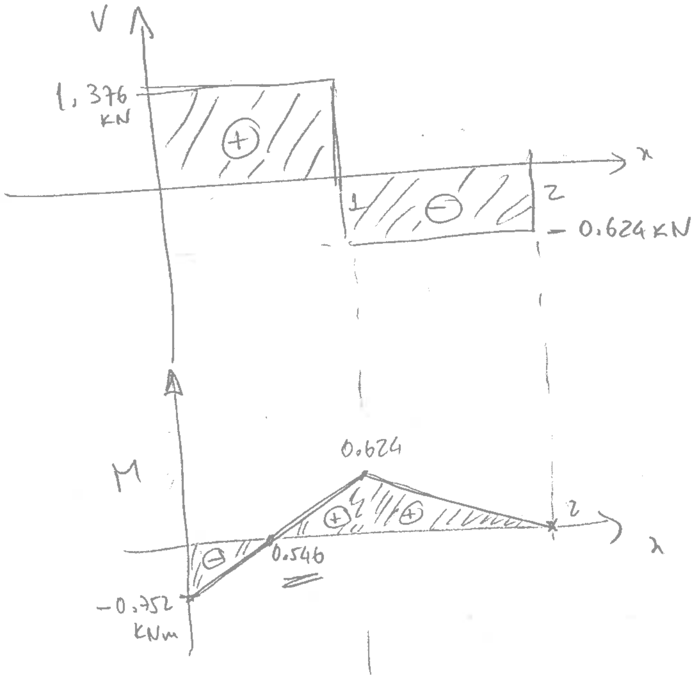
$V_{CB} - 1{,}376 + 2 = 0$

$V_{BC} = -2 + 1{,}376 = -0{,}624 \text{ kN}$

$M_{CB} + 0{,}752 - 1{,}376x + 2 \times (x-1) = 0$

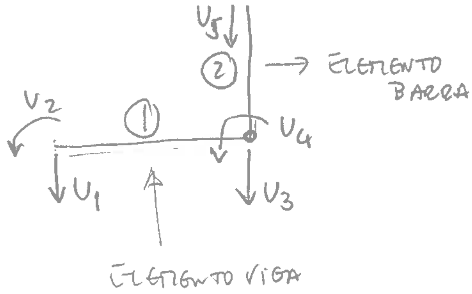
$M_{CB} + 0{,}752 - 1{,}376x + 2x - 2 = 0$

$M_{CB} = -0{,}624x + 1{,}248 \text{ kNm}$



PROBLEMA IV

a)



	1	2	3	4
1	1	2	3	4
2	5	3		

NOTA: → ELEMENTO VIGA

$$[K]_{VIGA} = \frac{2EI}{h^3}\begin{bmatrix} 6 & -3h & -6 & 3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix}$$

→ ELEMENTO BARRA

$$[K]_{BARRA} = \frac{EA}{h}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

b) $h=2$

$$[K]_{VIGA} = \frac{2\times 70\times 10^9 \times 5\times 10^{-6}}{2^3}\begin{bmatrix} 6 & -3\times 2 & -6 & 3\times 2 \\ -3\times 2 & 2\times 2^2 & 3\times 2 & 2^2 \\ -6 & 3\times 2 & 6 & 3\times 2 \\ -3\times 2 & 2^2 & 3\times 2 & 2\times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 5{,}25 & -5{,}25 & 5{,}25 & 5{,}25 \\ 5{,}25 & 7{,}00 & 5{,}25 & 3{,}50 \\ -5{,}25 & 5{,}25 & 5{,}25 & 5{,}25 \\ 5{,}25 & 3{,}50 & 5{,}25 & 7{,}00 \end{bmatrix}\times 10^5$$

$$[K]_{BARRA} = \begin{bmatrix} 350 & -350 \\ -350 & 350 \end{bmatrix}\times 10^5$$

K GLOBAL

$$10^5 \times \begin{bmatrix} 5{,}25 & 5{,}25 & 5{,}25 & 5{,}25 & 0 \\ 5{,}25 & 7{,}00 & 5{,}25 & 3{,}50 & 0 \\ -5{,}25 & 5{,}25 & 5{,}25+350 & 5{,}25 & -350 \\ 5{,}25 & 3{,}50 & 5{,}25 & 7{,}00 & 0 \\ 0 & 0 & -350 & 0 & 350 \end{bmatrix}$$

$K^G_{11} = K^V_{11}$ $K^G_{12} = K^V_{12}$ $K^G_{13} = K^V_{13}$ $K^G_{14} = K^V_{14}$ $K^G_{15} = 0$

$K^G_{22} = K^V_{22}$ $K^G_{23} = K^V_{23}$ $K^G_{24} = K^V_{24}$ $K^G_{25} = 0$

$K^G_{33} = K^V_{33} + K^B_{22}$ $K^G_{34} = K^V_{34}$ $K^G_{35} = K^B_{12}$

$K^G_{44} = K^V_{44}$ $K^G_{45} = 0$

$K^G_{55} = K^B_{22}$

c) $F_{VIGA} = \frac{qh}{12}\begin{Bmatrix} 6 \\ -h \\ 6 \\ h \end{Bmatrix}$ $h=2 \Rightarrow f_{VIGA} = \frac{1000\times 2}{12}\begin{Bmatrix} 6 \\ -2 \\ 6 \\ 2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -0{,}333 \\ 1 \\ 0.333 \end{Bmatrix}\times 10^3$

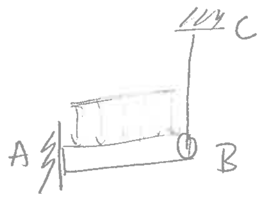
MMC
1ª Época
2010/2011
15/JAN/2011
7

c) cont.

$$F_{BARRA} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$F_{GLOBAL} = \begin{Bmatrix} 1 \\ -0{,}333 \\ 1 \\ 0{,}333 \\ 0 \end{Bmatrix} \times 10^3$$

$$F_1^G = F_1^{VIGA}$$
$$F_2^G = F_2^{VIGA}$$
$$F_3^G = F_3^{VIGA} + F_3^{BARRA}$$
$$F_4^G = F_4^{VIGA}$$
$$F_5^G = F_5^{BARRA}$$



+ Reacções.

d) EM A A DEFORMADE E ROTAÇÃO SÃO NULOS

LOGO $U_1 = U_2 = 0$

NO PONTO C (SUPORTE DA BARRA) O DESLOCAMENTO É NULO

LOGO $U_5 = 0$

$$10^5 \times \begin{bmatrix} 5{,}25 & 5{,}25 & 5{,}25 & 5{,}25 & 0 \\ 5{,}25 & 7{,}00 & 5{,}25 & 3{,}50 & 0 \\ -5{,}25 & 5{,}25 & 5{,}25+350 & 5{,}85 & -350 \\ 5{,}25 & 3{,}50 & 5{,}25 & 7{,}00 & 0 \\ 0 & 0 & -350 & 0 & 350 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -0{,}333 \\ 1 \\ 0{,}333 \\ 0 \end{Bmatrix} \times 10^3 + \begin{Bmatrix} R_A \\ M_A \\ 0 \\ 0 \\ R_C \end{Bmatrix}$$

FICAMOS $10^5 \times \begin{bmatrix} 350+5{,}25 & 5{,}25 \\ 5{,}25 & 7{,}00 \end{bmatrix} \begin{Bmatrix} U_3 \\ U_4 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0{,}333 \end{Bmatrix} \times 10^3$

$$355{,}25\, U_3 + 5{,}25\, U_4 = 1 \times 10^{-2}$$
$$5{,}25\, U_3 + 7\, U_4 = 0{,}333 \times 10^{-2}$$

$U_3 = 0{,}00002135$ m $= 0{,}214\ \mu m$

$U_4 = 0{,}0004597$ Rad $= 4{,}6 \times 10^{-6}$ Rad

$= 2{,}63 \times 10^{-2}$ GRAUS

e) $M = -EI \dfrac{d^2w}{dx^2}$

$$\frac{d^2w}{dx^2} = \sum_{i=1}^{4} U_i \frac{d^2\phi_i}{dx^2}$$

$$\phi_1 = 1 - 3\left(\frac{x}{h}\right)^2 + 2\left(\frac{x}{h}\right)^3$$

$$\frac{d\phi_1}{dx} = -6\frac{x}{h^2} + \frac{6}{h^3}x^2$$

$$\frac{d^2\phi_1}{dx^2} = -\frac{6}{h^2} + \frac{12}{h^3}x$$

$$\phi_2 = -x\left(1 - \frac{x}{h}\right)^2 = -x\left(1 - \frac{2x}{h} + \frac{x^2}{h^2}\right) = -x + \frac{2x^2}{h} - \frac{x^3}{h^2}$$

$$\frac{d\phi_2}{dx} = -1\left(+\frac{4x}{h}\right) - \frac{3x^2}{h^2}$$

$$\frac{d^2\phi_2}{dx^2} = \frac{4}{h} - \frac{6x}{h^2}$$

$$\phi_3 = 3\left(\frac{x}{h}\right)^2 - 2\left(\frac{x}{h}\right)^3$$

$$\frac{d\phi_3}{dx} = \frac{6x}{h^2} - \frac{6x^2}{h^3}$$

$$\frac{d^2\phi_3}{dx^2} = \frac{6}{h^2} - \frac{12x}{h^3}$$

$$\phi_4 = -x\left[\left(\frac{x}{h}\right)^2 - \frac{x}{h}\right] = -\frac{x^3}{h^2} - \frac{x^2}{h}$$

$$\frac{d\phi_4}{dx} = -\frac{3x^2}{h^2} - \frac{2x}{h}$$

$$\frac{d^2\phi_4}{dx^2} = -\frac{6x}{h^2} - \frac{2}{h}$$

Com $h = 2$

$$\frac{d^2\phi_1}{dx^2} = -\frac{3}{2} + \frac{3}{2}x \qquad \frac{d^2\phi_2}{dx^2} = 2 - \frac{3}{2}x$$

$$\frac{d^2\phi_3}{dx^2} = \frac{3}{2} - \frac{3}{2}x \qquad \frac{d^2\phi_4}{dx^2} = -\frac{3x}{2} - 1$$

$$M = -\cancel{EI} \times \left[0 \times \left(-\frac{3}{2} + \frac{3}{2}x\right) + 0 \times \left(2 - \frac{3}{2}x\right) + \frac{qL^4}{8\cancel{EI}}\left(\frac{3}{2} - \frac{3}{2}x\right) + \frac{qL^3}{6\cancel{EI}}\left(-\frac{3x}{2} - 1\right)\right]$$

$$M(x) = -2q\left(\frac{3}{2} - \frac{3}{2}x\right) - \frac{4}{3}q\left(-\frac{3x}{2} - 1\right) = -3q + 3qx + 2qx + \frac{4}{3}q$$

$L = 2$

$$= 5qx - \frac{5}{3}q$$

$$\boxed{M(x) = 5000x - \frac{5000}{3}}$$

A distribuição é <u>linear</u> quando a solução exacta conduz a distribuição <u>quadrática</u>.