MECÂNICA COMPUTACIONAL - ENG. BIOMÉDICA
ANO LECTIVO 2006/2007
3/JULHO/2007 1ª ÉPOCA

PROBLEMA I

a) $\sigma = \frac{F}{A}$ $\quad F = 110\ N$

$A = 12 \times 10 = 120\ mm^2 = 120 \times 10^{-6} m^2$

$\sigma = \frac{110}{120 \times 10^{-6}} = 916{,}66 \times 10^3\ Pa = 0{,}916\ MPa$ (compressão)
$= 916\ KPa$

b) Depois do ~~IMPLANTE~~

$F_{TOTAL} = F_{OSSO} + F_{IMPLANTE}$ → PROBL. ESTATICAMENTE INDETERMINADO

TEMOS DE UTILIZAR UMA EQ. ADICIONAL (DESLOCAMENTOS)

$\delta_{OSSO} = \delta_{IMPLANTE}$

$\frac{F_{OSSO} L_{OSSO}}{A_{OSSO} E_{OSSO}} = \frac{F_{IMP} L_{IMP}}{A_{IMP} E_{IMP}}$

COMO $L_{OSSO} = L_{IMP}$

$F_{OSSO} = \frac{A_{OSSO} E_{OSSO}}{A_{IMP} E_{IMP}} F_{IMP}$

E $F_{IMP} = F_{TOTAL} - F_{OSSO}$

$F_{OSSO} = \frac{A_{OSSO} E_{OSSO}}{A_{OSSO} E_{OSSO} + A_{IMP} E_{IMP}} F_{TOTAL}$

$A_{OSSO} = 120 \times 10^{-6} - \pi \times (0.002)^2 = 107{,}43 \times 10^{-6} m^2$

$A_{IMP} = 12{,}57 \times 10^{-6} m^2$

$E_{OSSO} = 10 \times 10^9\ Pa \quad E_{IMP} = 100 \times 10^9\ Pa$

$F_{OSSO} = 0.4622 \times 110 = 50.84\ N$

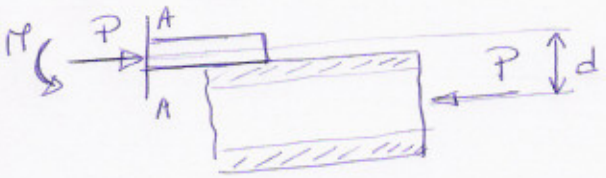
LOGO $\sigma_{OSSO} = \frac{F_{OSSO}}{A_{OSSO}} = \frac{50.84}{107{,}43 \times 10^{-6}} = 473{,}26 \times 10^3\ Pa = 473{,}26\ KPa$ (COMPRESSÃO)

c) $\delta = \frac{FL}{EA} = \frac{110 \times 0.015}{10 \times 10^9 \times 120 \times 10^{-6}} = 1.375 \times 10^{-6} m = 1.375 \mu m$

d) $\delta = \frac{FL}{EA} = \frac{50.84 \times 0.015}{10 \times 10^9 \times 107{,}43 \times 10^{-6}} = 0.710 \times 10^{-6} m = 0.710 \mu m$

e) O OSSO APÓS O IMPLANTE FICA SUJEITO A TENSÕES E DEFORMAÇÕES MENORES QUE PODEM LEVAR À ABSORÇÃO ÓSSEA E CONSEQUENTE PERDA DE MASSA ÓSSEA.

PROBLEMA II

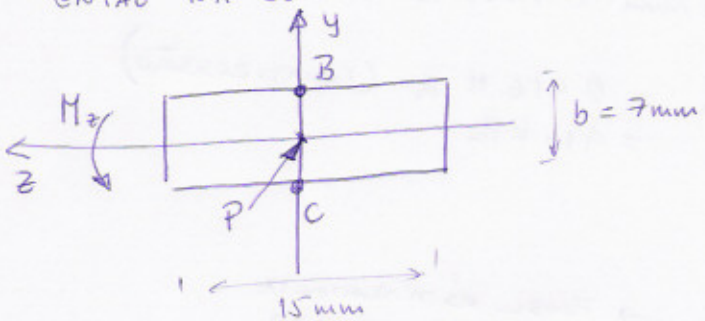


$M = P \times d = 1000 \times (12,5\times10^{-3} + 3,5\times10^{-3}) =$

$= 16\ Nm$

$P = 1000\ N$

ENTÃO NA SECÇÃO A-A



$M_z = -16\ Nm$

$P = -1000\ N$

$I_z = \frac{1}{12} \times 15\times10^{-3} \times (7\times10^{-3})^3 =$

$= 4,2875 \times 10^{-10}\ m^4$

$A = 15\times10^{-3} \times 7\times10^{-3} = 105 \times 10^{-6}\ m^2$

$$\sigma_x = \frac{P}{A} - \frac{M_z\, y}{I_z}$$

a) PONTO B $\quad y = +3,5\times10^{-3}\ m$

$$\sigma_x = -\frac{1000}{105\times10^{-6}} - \frac{(-16)\cdot(3.5\times10^{-3})}{4,2875\times10^{-10}} = 121,08\ MPa \quad (\text{TRACÇÃO})$$

b) PONTO C $\quad y = -3,5\times10^{-3}\ m$

$$\sigma_x = -\frac{1000}{105\times10^{-6}} - \frac{(-16)(-3.5\times10^{-3})}{4,2875\times10^{-10}} = -140,14\times10^6\ Pa$$

$$= -140,14\ MPa \quad (\text{COMPRESSÃO})$$

c) O CASO MAIS DESFAVORÁVEL É QUANDO A CONTRIBUIÇÃO DA CARGA AXIAL TEM O MESMO SINAL DO QUE A CONTRIBUIÇÃO DE FLEXÃO.

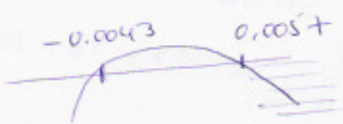
LOGO

$$\frac{1000}{0.015\times b} + \frac{1000\left(12,5\times10^{-3} + \frac{b}{2}\right)\times\frac{b}{2}}{\frac{1}{12}\times0.015\times b^3} \leq 200\times10^6$$

$$(266,666\times10^3)\,b + 5\times10^3 - 200\times10^6\, b^2 \leq 0$$

AS RAÍZES SÃO $\alpha = 0.0057$

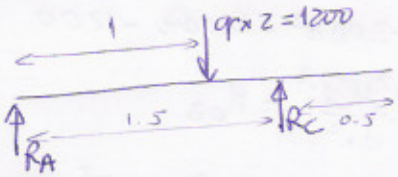
$\beta = -0.0043$



$b \geq 0.0057\ m = 5,7\ mm$

PROBLEMA III

a)



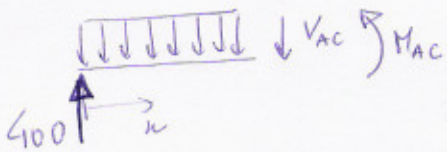
$\uparrow \sum F = 0 \quad R_A + R_C - 1200 = 0$

$+) \sum M_A = 0 \quad 1.5 R_C - 1200 \times 1 = 0$

$R_C = 800\ N$

$R_A = 400\ N$

b) AC



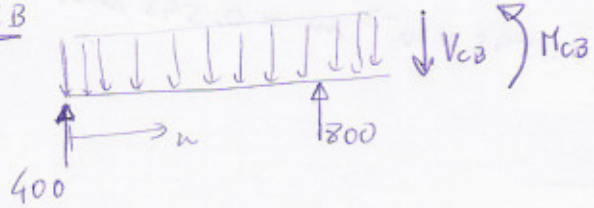
$V_{AC} - 400 + 600x = 0$

$V_{AC} = 400 - 600x$

$M_{AC} - 400x + 600x \cdot \frac{x}{2} = 0$

$M_{AC} = -300x^2 + 400x$

CB



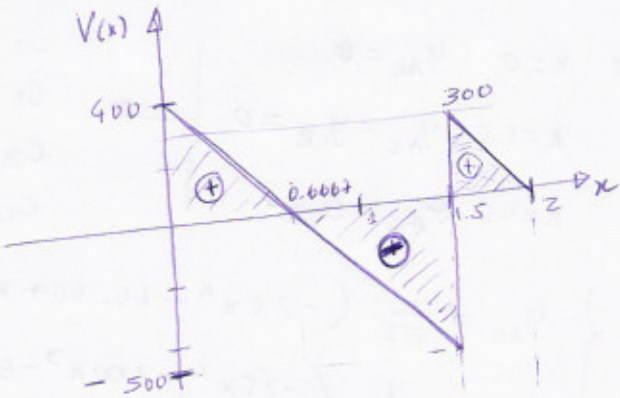
$V_{CB} - 400 - 800 + 600x = 0$

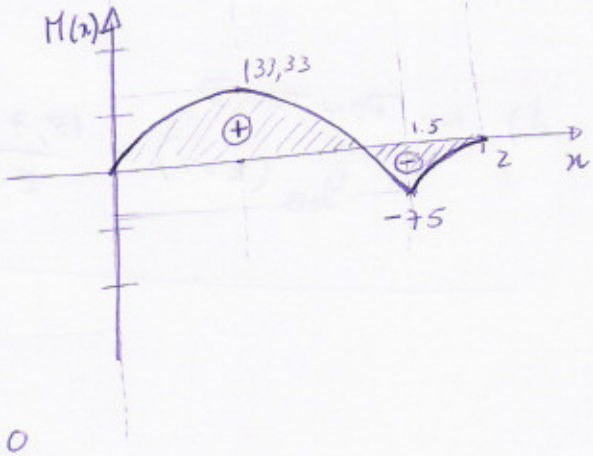
$V_{CB} = 1200 - 600x$

$M_{CB} - 400x - 800(x - 1.5) + 600x \frac{x}{2} = 0$

$M_{CB} = -300x^2 + 400x + 800x - 1200$

$M_{CB} = -300x^2 + 1200x - 1200$





c)

AC

$M_{AC} = -300x^2 + 400x$

$EI \frac{d^2 y_{AC}}{dx^2} = M_{AC}$

$EI \frac{dy_{AC}}{dx} = -300\frac{x^3}{3} + 400\frac{x^2}{2} + C_1$

$= -100x^3 + 200x^2 + C_1$

$EI\, y_{AC} = -100\frac{x^4}{4} + 200\frac{x^3}{3} + C_1 x + C_2$

$EI\, y_{AC} = -25x^4 + 66.667x^3 + C_1 x + C_2$

CB

$M_{CB} = -300x^2 + 1200x - 1200$

$EI \frac{d^2 y_{CB}}{dx^2} = M_{CB}$

$EI \frac{dy_{CB}}{dx} = -\frac{300x^3}{3} + 1200\frac{x^2}{2} - 1200x + C_3$

$= -100x^3 + 600x^2 - 1200x + C_3$

$EI\, y_{CB} = -100\frac{x^4}{4} + 600\frac{x^3}{3} - 1200\frac{x^2}{2} + C_3 x + C_4$

$EI\, y_{CB} = -25x^4 + 200x^3 - 600x^2 + C_3 x + C_4$

En $x = 0 \quad y_{AC} = 0$

$x = 1.5 \quad y_{AC} = y_{CB} = 0$

$x = 1.5 \quad \theta_{AC} = \theta_{CB}$

$\Rightarrow$

$C_2 = 0$

$C_1 = -65.625$

$C_3 = 834.375$

$C_4 = -450$

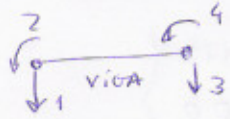
$$\begin{cases} y_{AC} = \frac{1}{EI}\left(-25x^4 + 66.667x^3 - 65.625x\right) & 0 < x < 1.5 \\ y_{CB} = \frac{1}{EI}\left(-25x^4 + 200x^3 - 600x^2 + 834.375x - 450\right) & 1.5 < x < 2 \end{cases}$$

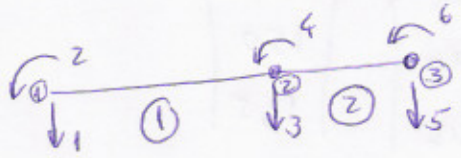
d) NO PUNTO D

$y_{CB}(x=2) = \frac{18,75}{EI} = 0.595 \times 10^{-3}\ m = 0.595\ mm$

PROBLEMA IV

a) ELEMENTO VIGA – 2 ELEMENTOS





b) $K^{①} = \frac{2EI}{1.5^3}$

6	-4.5	-6	-4.5
-4.5	4.5	4.5	2.25
-6	4.5	6	4.5
-4.5	2.25	4.5	4.5

$= EI \times$

3.5556	-2.6667	-3.5556	-2.6667
-2.6667	2.6667	2.6667	1.3333
-3.5556	2.6667	3.5556	2.6667
-2.6667	1.3333	2.6667	2.6667

$K^{②} = EI$

96	-24	-96	-24
-24	8	24	4
-96	24	96	24
-24	4	24	8

MATRIZ DE CONECTIVIDADES

	1	2	3	4
①	1	2	3	4
②	3	4	5	6

$K^{⑤} = EI$

3.5556	-2.6667	-3.556	-2.6667	0	0
-2.6667	2.6667	2.6667	1.3333	0	0
-3.556	2.6667	99.5556	-21.3333	-96	24
-2.6667	1.3333	-21.3333	10.6667	24	4
0	0	-96	24	96	24
0	0	24	4	24	8

c) $F^{①} = \frac{600 \times 1.5}{12} \begin{Bmatrix} 6 \\ -1.5 \\ 6 \\ -1.5 \end{Bmatrix} = \begin{Bmatrix} 450 \\ -112.5 \\ 450 \\ 112.5 \end{Bmatrix}$

$F^{②} = \frac{600 \times 0.5}{12} \begin{Bmatrix} 6 \\ -0.5 \\ 6 \\ 0.5 \end{Bmatrix} =$

$F^{⑤} = \begin{Bmatrix} 450 \\ -112.5 \\ 600 \\ 100 \\ 150 \\ 12.5 \end{Bmatrix}$

d) C.F. $u_1 = 0$; $u_3 = 0$

FICA UM SISTEMA 4x4

$$31{,}5 \times 10^3 \times \left[\begin{array}{c|c|c|c} 2.6667 & 1.3333 & 0 & 0 \\ \hline 1.3333 & 10.6667 & 24 & 4 \\ \hline 0 & 24 & 96 & 24 \\ \hline 0 & 4 & 24 & 8 \end{array}\right] \left\{\begin{array}{c} u_2 \\ u_4 \\ u_5 \\ u_6 \end{array}\right\} = \left\{\begin{array}{c} -112 \\ 100 \\ 150 \\ 12{,}5 \end{array}\right\}$$

$\Rightarrow$ $u_2 = -0.002075$

$u_4 = 0.001484$

$\boxed{u_5 = -0.000593}$ → deslocamento vertical

$u_6 = 0.00108$

0,59 mm ↑

e) NO MÉTODO DOS ELEMENTOS FINITO $y(x)$ É LINEAR

É PORTANTO

$$M = EI \frac{d^2y}{dx^2} \quad \text{É LINEAR}$$

ENQUANTO QUE NA SOLUÇÃO EXACTA O MOMENTO FLECTOR É QUADRÁTICO COMO SE VIU NA ALÍNEA b) LOGO O CÁLCULO DO MOMENTO PELO MEF NÃO É EXACTO.