EXAME 1ª ÉPOCA - MEC. COMPUTACIONAL - ENG. BIOMÉDICA 3/JULHO/200

PROBLEMA I

a) tensão no osso.

O osso está à compressão e a tensão é

$$\sigma = \frac{F}{A} \quad \text{com Area do osso} \quad A = \pi\left((12.5\times10^{-3})^2 - (7.5\times10^{-3}\right.$$
$$= 3.1416\times10^{-4}$$

$$\sigma = -\frac{1000}{3.1416\times10^{-4}} = -3.183\times10^{6}\,Pa$$
$$= -3.183\ MPa$$

b) A tensão normal em cada uma das placas é

$$\sigma = \frac{F_{PLACA}}{A_{PLACA}} \qquad f_{PLACA} = F/2$$

$$\sigma = -\frac{500}{9.5\times10^{-5}} = -5.26\times10^{6} = -5.26\ MPa$$

c) A tensão de corte nos parafusos.

$$\tau = \frac{F/2}{A_{parafuso}} = \frac{500}{\pi(2.5\times10^{-3})^2} = 25.46\times10^{6}\,Pa = 25,46\ MPa$$

d) A deformação axial das placas é

$$\delta = \frac{PL}{EA} = \frac{500\times(50\times10^{-3})}{200\times10^{9}\times9.5\times10^{-5}} = 1.316\times10^{-6}\,m$$
$$= 1.316\,\mu m$$

PROBLEMA 2

a) A tensão no ponto A é 11.2 MPa de compressão, logo:

$$-11.2\times10^{6} = -\frac{P}{A} - \frac{My}{I} \quad \text{com} \quad P = 1000N$$

$A = \pi(r_{ext}^2 - r_{int}^2)\ m^2$

$M = 1000\times25\times10^{-3}\ Nm$

$y = r_{ext} = 16\times10^{-3}\,m$

$I = \frac{\pi}{4}(r_{ext}^4 - r_{int}^4)$

$$-11.2\times10^{6} = -\frac{1000}{\pi(r_{ext}^2 - r_{int}^2} - \frac{1000\times25\times10^{-3}\times16\times10^{-3}}{\frac{\pi}{4}(r_{ext}^4 - r_{int}^4)}$$

$$-11.2\times10^{6}(r_{ext}^4 - r_{int}^4) = -\frac{1000}{\pi}(r_{ext}^2 + r_{int}^2) - \frac{1000\times16\times10^{-3}\times25\times10^{-3}}{\pi/4}$$

$$-11.2\times10^{6}\,r_{ext}^4 + 11.2\times10^{6}\,r_{int}^4 = -318.31\,r_{ext}^2 - 318.31\,r_{int}^2 - 0.509$$

$$11.2\times10^{6}\,r_{int}^4 + 318.31\,r_{int}^2 - 0.1432 = 0$$

$$11.2\times10^{6}x^{2}+318.31x-0.1432=0$$

CUJAS RAÍZES SÃO:

$$x=-128.174\times10^{-6} \quad \text{ou} \quad x=99.7531\times10^{-6}$$

A RAIZ NEGATIVA NÃO PODE SER LOGO

$$r_{int}^{2}=99.7531\times10^{-6} \Rightarrow r=9.987\times10^{-3}\ mm$$

$$d_{int}=19.975\ mm \simeq 20\ mm$$

A espessura do osso compacto é:

$$e=\frac{32-20}{2}=\underline{\underline{6\ mm}}$$

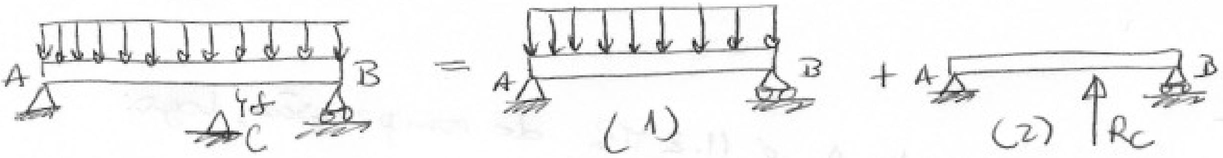
b) com $d_{int}=20\ mm$

$$\sigma_B=-\frac{1000}{\pi(r_{ext}^{2}-r_{int}^{2})}+\frac{1000\times25\times10^{-3}\times16\times10^{-3}}{\frac{\pi}{4}(r_{ext}^{4}-r_{int}^{4})}=7.13\times10^{6}\ Pa =7.13\ MPa$$

c) $$\sigma_C=-\frac{1000}{\pi(r_{ext}^{2}-r_{int}^{2})}=-2.04\times10^{6}\ Pa=-2.04\ MPa$$

PROBLEMA 3

a) O PROBLEMA É ESTATICAMENTE INDETERMINADO. UTILIZANDO O MÉTODO DA SOBREPOSIÇÃO E COM A AJUDA DAS TABELAS DO FORMULÁRIO TEMOS:



$$e \quad y_{(1)}+y_{(2)}=\delta$$

PROBLEMA (1)

$$y=-\frac{q}{24EI}(x^{4}-2Lx^{3}+L^{3}x)$$

$$y(x=1.4)=-\frac{600}{24EI}(1.4^{4}-2\times2\times1.4^{3}+2^{3}\times1.4)=-32{,}26\times10^{-3}\ m$$

PROBLEMA (2)

$$y(x=1.4)=\frac{R_C\times1.4^{2}\times0.6^{2}}{3\times70\times10^{9}\times45\times10^{-9}\times2}=3.73\times10^{-5}R_C$$

COMO $y_{(1)}+y_{(2)}=\delta=-10\times10^{-3} \Rightarrow -32{,}26\times10^{-3}+3{,}73\times10^{-5}R_C=-10\times10^{-3}$

$$\Rightarrow \underline{R_C=596{,}43\ N}$$

OUTRAS REACÇÕES SÃO OBTIDAS DAS EQS. DA ESTÁTICA,

$\uparrow\Sigma F=0 \quad R_A+R_B+596{,}43-600\times2=0$

$\circlearrowright\Sigma M_A=0 \quad 596{,}43\times1.4+R_B\times2-600\times2\times1=0$

$$\Bigg\} \quad \underline{R_B=182{,}50\ N} \quad \underline{R_A=421{,}07\ N}$$

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b) A deformada pode ser determinada utilizando as tabelas do formulário:

Entre AC

$$y(x) = -\frac{R_C\, b}{6EIL}\left[x^3 - (L^2 - b^2)x\right] - \frac{q}{24EI}\left(x^4 - 2Lx^3 + L^3 x\right)$$

com $b = 0.6\ m$

$L = 2\ m$

$R_C = 594,93\ N$

$q = 600\ N/m$

Entre CB

$$y(x) = -\frac{R_C\, a}{6EIL}\left[(L-x)^3 - (L^2 - b^2)(L-x)\right] - \frac{q}{24EI}\left(x^4 - 2Lx^3 + L^3 x\right)$$

com $a = 1.4\ m$

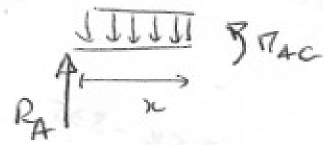
Logo y a meio da viga é:

$$y_{AC}(x=1) = -\frac{596,43 \times 0.6}{6 \times 70\times10^9 \times 45\times10^{-9} \times 2}\left[1 - (2^2 - 0.6^2)\times 1\right] - \frac{600}{24\times70\times10^9\times45\times10^{-9}}\left(1 - 2\times2 + 2^3\right)$$

$$y_{AC}(x=1) = -14,68\times10^{-3}\ m = \underline{\underline{-14,68\ mm}}$$

Ou então utilizando integração a partir da expressão dos momentos:

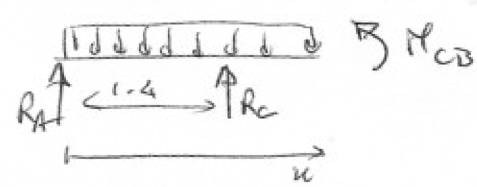
AC



$$M_{AC} - 421,07\,x + 600\frac{x^2}{2} = 0$$

$$M_{AC} = -300x^2 + 421,07x$$

CB



$$M_{CB} + 600\frac{x^2}{2} - 421,07x - 596,43(x - 1.4) = 0$$

$$M_{CB} = -300x^2 + 1017.5\,x - 835$$

Integrando 2 vezes

AC

$$EI\frac{dy_{AC}}{dx} = -300\frac{x^3}{3} + 421,07\frac{x^2}{2} + C_1$$

$$EI\,y_{AC} = -300\frac{x^4}{12} + 421,07\frac{x^3}{6} + C_1 x + C_2$$

$$EI\frac{dy_{AC}}{dx} = -100x^3 + 210,53x^2 + C_1$$

$$EI\,y_{AC} = -25x^4 + 70,18x^3 + C_1 x + C_2$$

CB

$$EI\frac{dy_{CB}}{dx} = -300\frac{x^3}{3} + 1017,5\frac{x^2}{2} - 835x + C_3$$

$$EI\,y_{CB} = -300\frac{x^4}{12} + 1017,5\frac{x^3}{6} - 835\frac{x^2}{2} + C_3 x + C_4$$

$$EI\frac{dy_{CB}}{dx} = -100x^3 + 508,75x^2 - 835x + C_3$$

$$EI\,y_{CB} = -25x^4 + 169,58x^3 - 417,5x^2 + C_3 x + C_4$$

COND. FRONTEIRA

$x=0 \quad y_{AC}=0 \Rightarrow C_2=0$

$x=2 \quad y_{CB}=0 \Rightarrow -713,36+2C_3+C_4=0$

COMPATIBILIDADE EM $x=1.4$

$$\frac{dy_{AC}}{dx}=\frac{dy_{CB}}{dx} \Rightarrow 584,48+C_1-C_3=0$$

$$y_{AC}=y_{CB} \Rightarrow 545,54+1.4C_1-1.4C_3-C_4=0$$

$$\Rightarrow \begin{cases} C_1=-91,86 \\ C_2=0 \\ C_3=492,62 \\ C_4=-272,76 \end{cases}$$

então $y_{AC}=\frac{1}{EI}\left(-25x^4+70,18x^3-91,86x\right)$

e $y_{CB}=\frac{1}{EI}\left(-25x^4+169,58x^3-417.5x^2+492,62x-272,76\right)$

e $y_{AC}(x=1)=-14.68\times10^{-3}\ m=-14.68\ mm$

c) Diagrama de V e M

AB $\quad V_{AC}$, R_A, x

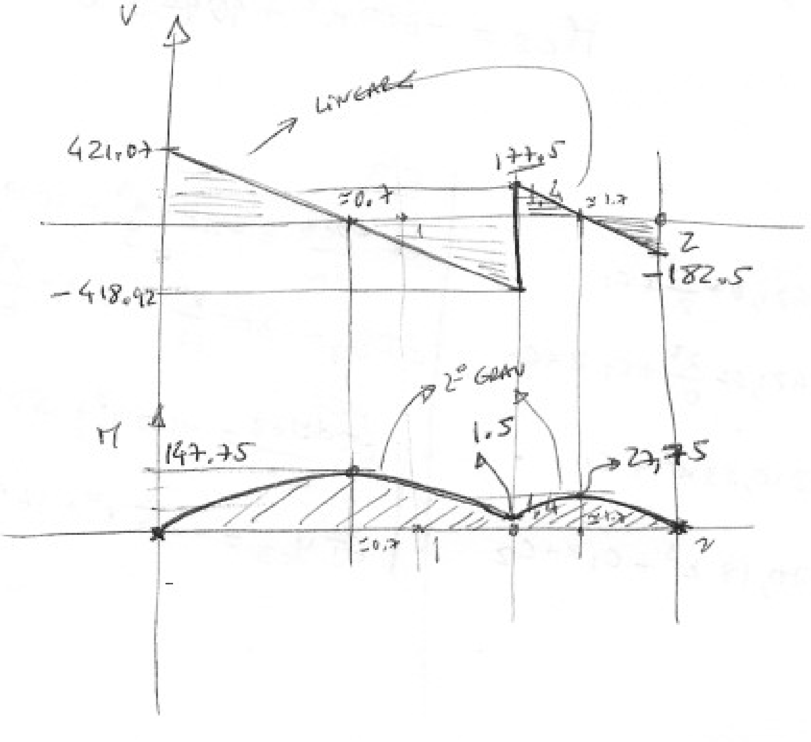
$V_{AB}+600x-421,07=0$

$V_{AB}=-600x+421,07 \qquad M_{AB}=-300x^2+421,07x$

CB $\quad V_{BC}$, R_A, 1.4, R_C, x

$V_{CB}=-600x+596,43+421,07$

$V_{CB}=-600x+1017.5 \qquad M_{CB}=-300x^2+1017.5x-835$



PROBLEMA 4

EXAME 1ª ÉPOCA
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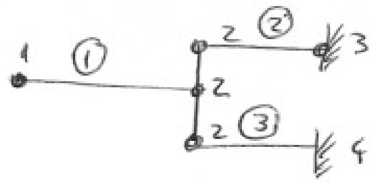

→ elemento linear unidimensional (elemento barra)

ELEMENTO	$A^{(1)}$	$B^{(2)}$
①	1	2
②	2	3
③	2	4

a) $$K^{①} = \frac{E^{①} A^{①}}{L^{①}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$E^{①} = 20\,GPa$

$A^{①} = 3.1416 \times 10^{-4}\, m^2$

$L^{①} = 0{,}1\, m$

$$K^{①} = \begin{bmatrix} 62.83 & -62.83 \\ -62.83 & 62.83 \end{bmatrix} \times 10^6$$

$$K^{②} = K^{③} = \frac{E^{②} \times A^{②}}{L^{②}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$E^{②} = E^{③} = 200\,GPa$

$A^{②} = A^{③} = 9.5 \times 10^{-5}\, m^2$

$L^{②} = 25 \times 10^{-3}\, m$

$$K^{②} = K^{③} = \begin{bmatrix} 760 & -760 \\ -760 & 760 \end{bmatrix} \times 10^6$$

b) A MATRIZ GLOBAL É 4×4

$K_{11}^{G} = k_{11}^{①}$ $\quad K_{12}^{G} = k_{12}^{①}$ $\quad k_{13}^{G} = 0$ $\quad K_{14}^{G} = 0$

$K_{22}^{G} = k_{11}^{②} + k_{11}^{③} + k_{22}^{①}$ $\quad K_{23}^{G} = k_{12}^{②}$ $\quad K_{24}^{G} = K_{12}^{③}$

$K_{33}^{G} = k_{22}^{②}$ $\quad K_{34}^{G} = 0$

$K_{44}^{G} = k_{22}^{③}$

$$K^{GLOBAL} = \begin{bmatrix} 62.83 & -62.83 & 0 & 0 \\ -62.83 & \begin{matrix} 2\times 760 \\ +62.83 \end{matrix} & -760 & -760 \\ 0 & -760 & 760 & 0 \\ 0 & -760 & 0 & 760 \end{bmatrix} \times 10^6$$

$K_{22}^{G} = 2 \times 760 + 62.83 = \underline{1582{,}83}$

c) Não existem forças distribuídas

forças concentradas só temos no nó 1

$$F^G = \begin{Bmatrix} 1000 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

d) $u_3 = u_4 = 0$

$$10^6 \times \begin{bmatrix} 62.83 & -62.83 & 0 & 0 \\ -62.83 & 1582.83 & -760 & -760 \\ 0 & -760 & 760 & 0 \\ 0 & -760 & 0 & 760 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 1000 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Como u_3 e $u_4 = 0$ posso eliminar as linhas e colunas correspondente e fica

$$10^6 \times \begin{bmatrix} 62.83 & -62.83 \\ -62.83 & 1582.83 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 1000 \\ 0 \end{Bmatrix}$$

e) Resolvendo o sistema

$$\begin{cases} 10^6 \times (62.83\, u_1 - 62.83\, u_2) = 1000 \\ 10^6 \times (-62.83\, u_1 + 1582.83\, u_2) = 0 \end{cases}$$

$$\Rightarrow \quad u_1 = 16.57 \times 10^{-6}\ m$$
$$u_2 = 6.57 \times 10^{-7}\ m \rightarrow \delta \text{ de meia placa}$$

para a placa inteira $\delta = u_2 \times 2 = 1,316 \times 10^{-6}\,m$

$= \underline{\underline{1,316\ \mu m}}$