Bone Tissue Mechanics

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PART 7

Bone Adaptation

Bone Modelling vs. Remodelling

<u>osteogenesis –</u> bone production from soft tissue (fibrosis tissue or cartilage). Bone formation in a early stage of growth. It also happens in bone healing.

<u>bone modelling</u> – Modelling results in change of bone size and shape. The rate of modelling is greatly reduced after skeletal matutity. Involves independent actions of osteoclasts and osteoblasts.

• ***<u>bone remodelling</u>*** – Process of replacement of "old bone" by "new bone". It repairs damage and prevent fatigue damage. Usually does not affect size and shape. Occurs throughout life, but is also substantially reduced after growth stops. A combined action of osteoclasts and osteoblasts (BMU - *Basic multicellular unit*).

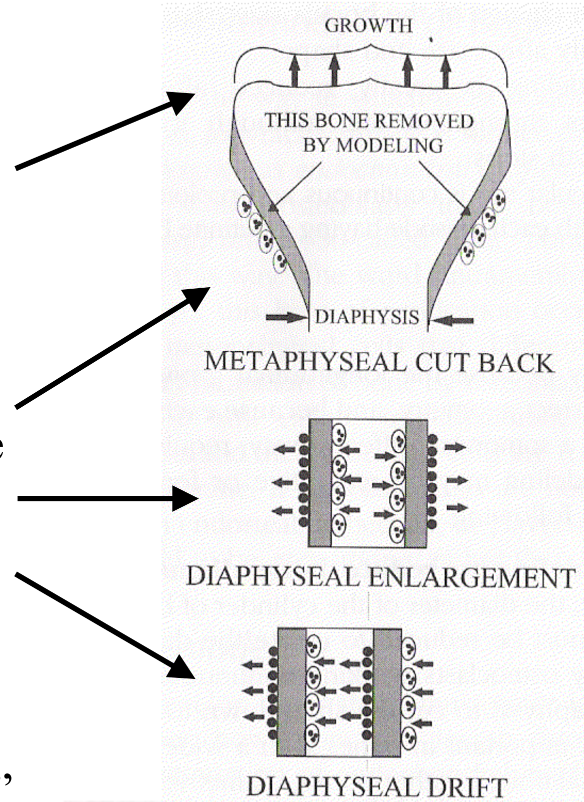


FIGURE 2.19. *Top:* Resorptive modeling beneath the growth plate to form the diaphysis from the metaphysis. *Middle:* Formative (periosteal surface) and resorptive (endosteal surface) modeling to enlarge the diaphysis. *Bottom:* Modeling to "drift" the diaphysis to the left (thereby altering diaphyseal curvature).

Bone adaptation (*“remodelling”*)

- *bone modelling*

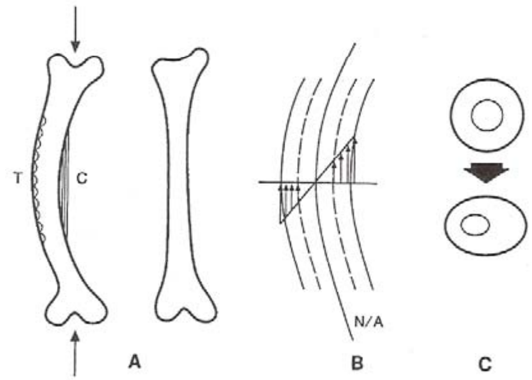
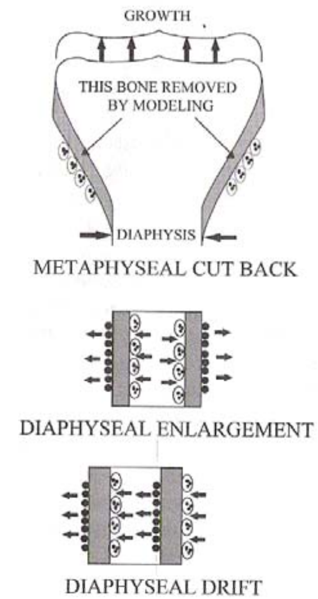


FIGURE 6.4A–C. When a fractured bone heals in an angulated position (A, *left*), bending stresses produced by end loads may provide the stimulus for straightening (A, *right*). However, the assumption that tensile stress (*T*) promotes resorption and compressive stress (*C*) promotes formation is too simple because the endosteal surfaces have similar stresses (B, longitudinal section through the bent bone, *N/A*, neutral axis). This concept results in resorption and formation on the left and right endosteal surfaces, respectively, producing an asymmetric cross section (C).



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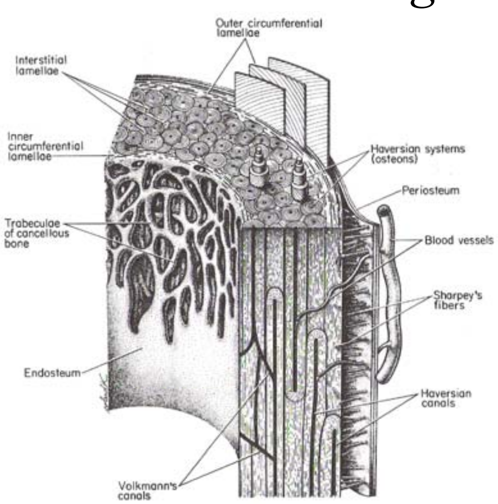
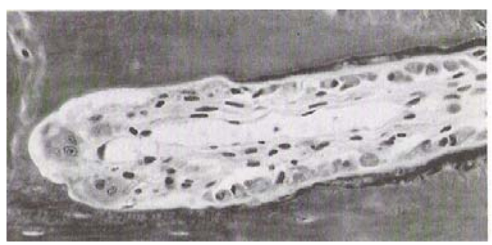
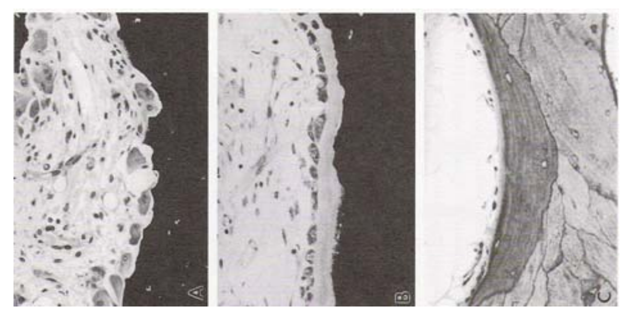
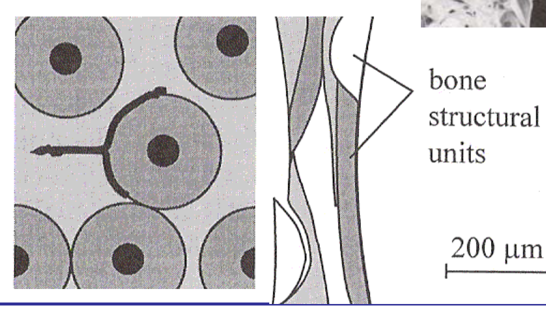
- *bone remodelling*









Bone adaptation

- Bone adapts depending on its function
  - this adaptation is not only in terms of density or trabecular orientation, but it affect all properties of bone.
  - the adaptation is not only the result of an abnormal situation

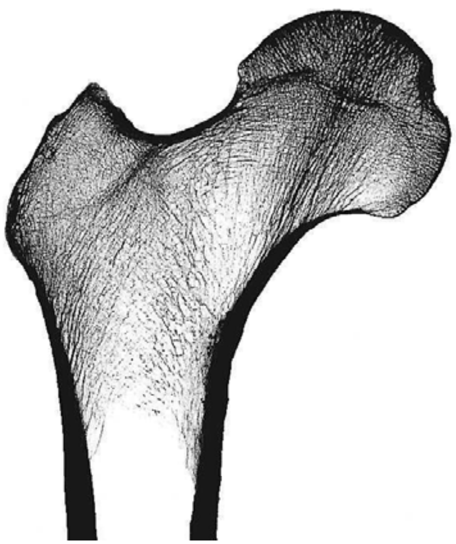
TABLE 6.1. Material properties of three different bones

Variable	Red deer antler	Bovine femur	Whale bulla
Work-of-fracture, J/m^2	6190	1710	200
Bending strength, MPa	179	247	33
Elastic modulus, GPa	7.4	13.5	31.3
Density, g/ml	1.86	2.06	2.47
Acoustic impedance, 10^9 kg m^{-2} s^{-1}	3.71	5.27	8.79
Mineral content, % by weight	59.3	66.7	86.4

After Currey, 1981.

Bone adaptation

• internal adaptation (internal remodeling)



• external adaptation (external remodeling) – surface adaptation)

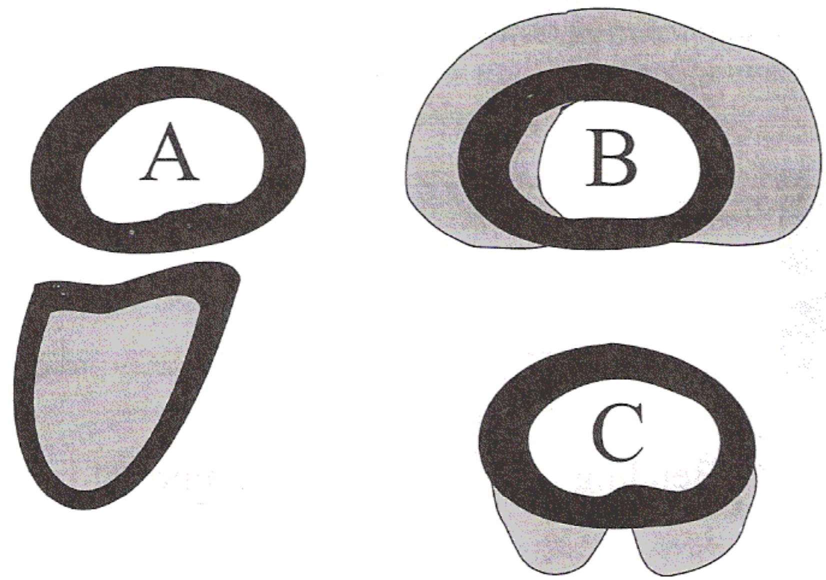


FIGURE 6.8. Results of osteotomy on young pigs. **A** Normal midshaft cross-sectional geometry of the radius and ulna (below) prior to surgical removal of the central ulnar diaphysis. Cortical bone is black; cancellous bone is gray. **B** Appearance of the radius a few weeks after surgery; here, gray material represents woven bone. **C** Three months after ulnar osteotomy the woven bone shown in B has disappeared and new woven bone has appeared facing the missing ulna. (Redrawn with permission from Goodship et al., 1979.)

Bone adaptation – historical perspective

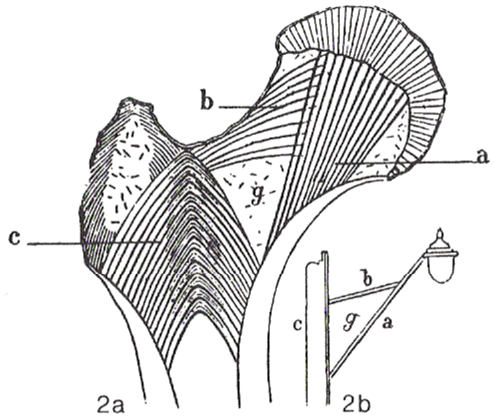


FIGURE 6.1. Sketch showing Ward's analogy between the trabecular arrangement in the neck of the femur and a streetlamp bracket. (From The laws of bone architecture, Koch, JC, *American Journal of Anatomy*, 1917. Reprinted by permission from Wiley-Liss, Inc., a subsidiary of John Wiley & Sons, Inc.)

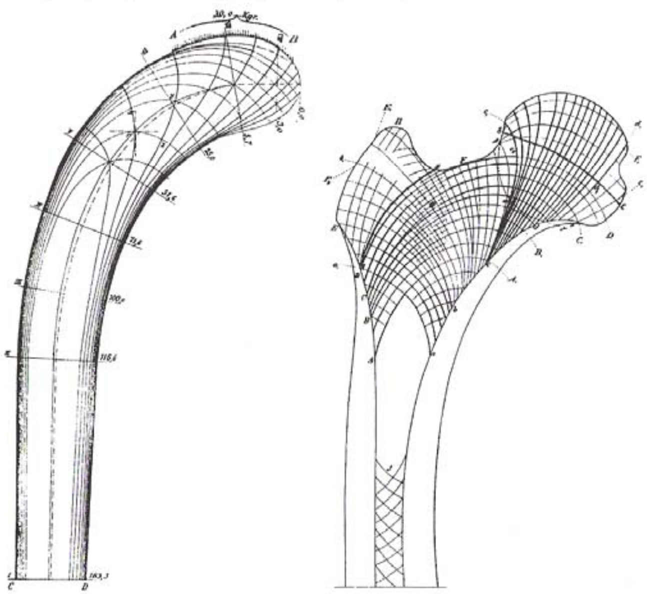
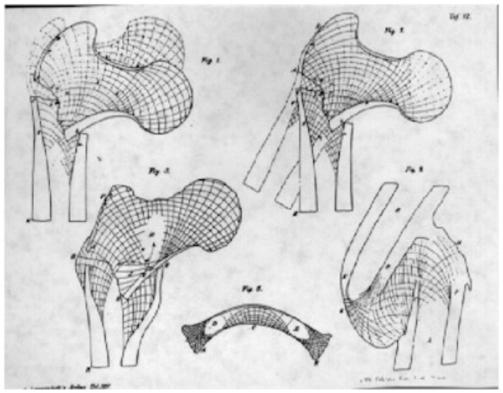


FIGURE 6.2. *Left:* Principal stress trajectories in a Fairbairn crane as calculated by Culmann. *Right:* The trabecular arrangement in a proximal human femur as sketched by von Meyer. (Reproduced with permission from Maquet and Furlong's translation of Wolff's 1892 book.)

• In 1838 Ward showed the analogy between the trabecular arrangement and a street lamp. (region *g* is represent less dense trabecular bone (sparse trabecular struts), usually known as Ward's triangle)

• In 1867 Von Meyer (anatomist) and Culmann (Engineer) observe that the trabeculae are oriented as the principal direction of stresses in a curved crane.

Bone adaptation – historical perspective



• In 1892 Wolff (anatomist) publish the book "*Das Gesetz der Transformation der Knochen" (The Law of Bone Remodeling)*, where his findings on bone physiology are collected. In this book, he state that: "*Every change in the form and the function of a bone or of their function alone is followed by certain definite changes in their internal architecture and equally definite secondary alterations in their external conformation, in accordance with mathematical laws*".

•Wolff's statements are know as the "Wolff's Law".
•Today, the meaning of Wolff's Law incorporate concepts behind of the Wolff's observation.
• The basic ideas of Wolff's observations are:

✓ Optimization of strength with respect to weight
✓ Alignment of trabeculae with principal stress directions
✓ Self-regulation of bone structure by cells responding to a mechanical stimulus.

Bone adaptation – historical perspective

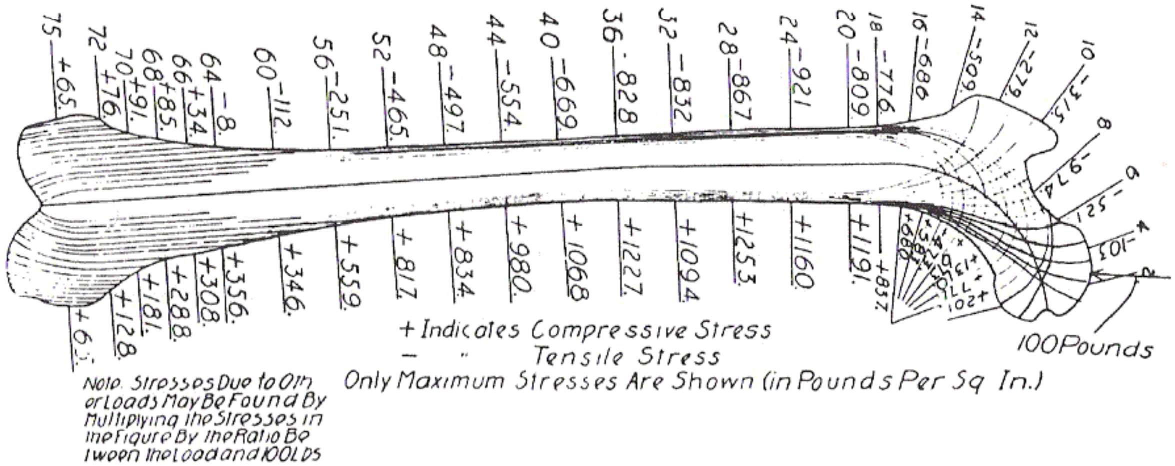


FIGURE 6.3. Koch's computation of the principal stress trajectories and values in a human femur. Note the applied load vector on the femoral head. Koch, 1917.

• In 1917, Koch confirm the *trajectorial theory*, i.e., the alignment of trabeculae with the stress principal directions. He suggests that the bone density is higher where the shear stress is maximum. He also suggests that bone reach the maximum of strength with a minimum of bone mass.

• By the end of 1920, there appears the "idea" that bone cells can response to local mechanical loads and proceed to the correspondent bone adaptation.

Bone adaptation – historical perspective

• Between 1938-1941, Glucksman performed *in-vitr*o experiments with tissues on different growing stages:

- subject the growing tissue to different stress levels.
- zones with higher tensile stresses are more ossified.
- ossifying tissue is aligned along the principal tensile stresses.

• In 60's, Frost (cirurgião ortopédico) studied the physiological mechanisms of bone adaptation:

- bone adaptation is the result of processes of modeling and remodeling.
- on remodeling the osteoclasts and osteoblasts work together without a significant bone mass change.
- the relation between strain and bone mass is different in the growing phase (modeling) and in the mature phase (remodeling)

Bone adaptation – historical perspective

- In 1972 Chamay e Tschantz performed osteotomy of the canine radius and observed:
    - at 9 weeks, significant hypertrophy with 60% to 100% increase in cortical thickness
    - Noted several cases of fatigue fracture
    - Carter estimated strains were between 5000 and 7000 μstrain; suggested hypertrophy due to damage

- In 1981 Carter et al. performed osteotomy of ulna in Canines.
    - Although the increased from 600 μstrain to 1500 μstrain after osteotomy, no significant change in bone geometry were observed.
    - It was assumed the existence of a *lazy zone* on bone adaptation. Basically a plateau on the strain level where the bone (in adults) does not adapt.

- The work of Rubin and Lanyon lead to a similar conclusions, i.e, for adults there is a homeostatic level of strain.

Bone adaptation – historical perspective

• Rubin and Lanyon (1982) did a survey of the peak strain values in various animal during their daily activity. Values are between 2000-3000 με.

TABLE 6.2. Peak functional strains in various animal bones

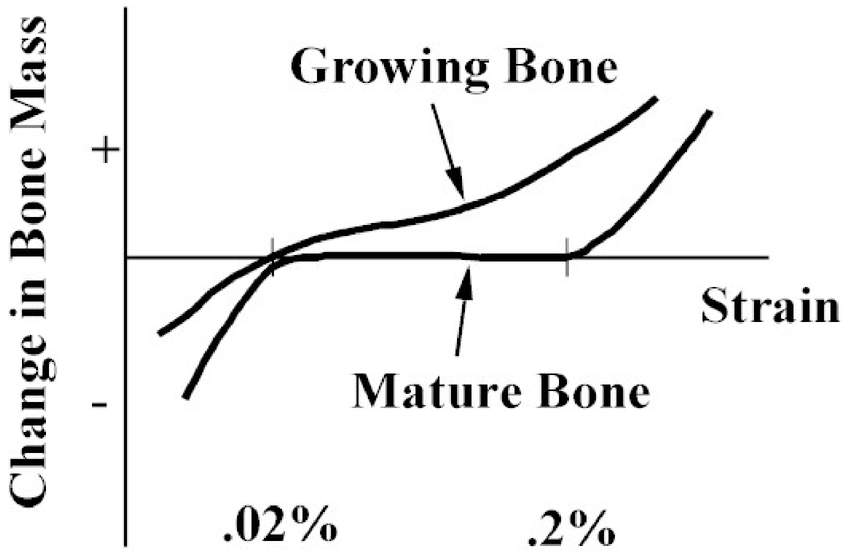
Bone	Activity	Peak strain	Reference
Horse radius	Trotting	−2800	Rubin and Lanyon, 1982
Horse tibia	Galloping	−3200	Rubin and Lanyon, 1982
Horse metacarpal	Acceleration	−3000	Biewener et al., 1983
Dog radius	Trotting	−2400	Rubin and Lanyon, 1982
Dog tibia	Galloping	−2100	Rubin and Lanyon, 1982
Goose humerus	Flying	−2800	Rubin and Lanyon, 1982
Cockerel ulna	Flapping	−2100	Rubin and Lanyon, 1982
Sheep femur	Trotting	−2200	Rubin and Lanyon, 1982
Sheep humerus	Trotting	−2200	Rubin and Lanyon, 1982
Sheep radius	Galloping	−2300	O'Conner et al., 1982
Sheep tibia	Trotting	−2100	Lanyon et al., 1982
Pig radius	Trotting	−2400	Goodship et al., 1979
Fish hypural	Swimming	−3000	Rubin and Lanyon 1982
Macaca mandible	Biting	−2200	Hylander, 1979
Turkey tibia	Running	−2350	Rubin and Lanyon, 1984

After Rubin and Lanyon, 1982.

• They also conclude the static load lead to resorption, and so, it is necessary dynamic loading to maintain bone. (4 cycles per day at 2050 με, is enough).

Bone adaptation – historical perspective

• Frost suggested a different bone adaptation behavior in adolescent and adults.
• Adolescents have a higher sensitivity to the mechanical stimulus than adults, because there are modeling and remodeling simultaneously. The lack of modeling in adults diminish the sensitivity to mechanical stimulus.



Bone adaptation – mathematical models

• In general, it is accepted that bone adaptation is a change on bone structure depending on the mechanical stimulus as well as on physiological mechanisms.

• Usually, bone density and trabecular orientation (for internal adaptation) and bone surface density (for external adaptation) are parameters used to define the bone structure.

•As mechanical stimulus, variables as strain, stress and strain energy density are often considered.

Bone adaptation – adaptive elasticity theory

•In 1976 Cowin *et al.* proposed a pioneer mathematical model. Based on continuum mechanics laws, he assumed bone as a poroelastic material and established the adaptive elasticity theory where the mechanical stimulus is the strain, $\boldsymbol{e}=e_{ij}$.

• The model considers external and internal adaptation, that can be written in a simplified way as:

External Adaptation – the bone boundary is changing until it reaches a stationary stage according to: (υ represents the remodeling velocity in the normal direction).

$$\upsilon(P) = C_{ij}(P,\boldsymbol{n})\cdot\left(e_{ij}(P) - e_{ij}^{0}(P)\right)$$

External Remodeling – the bone volume fraction changes until it reaches a stationary stage according to (μ is the volume fraction).

$$\frac{d\tilde{\mu}}{dt} = a(\tilde{\mu}) + A_{ij}(\tilde{\mu})e_{ij} + B_{ijkl}(\tilde{\mu})e_{ij}e_{kl}, \quad \tilde{\mu} = \mu - \mu_0$$

• Later, Cowin introduced the *fabric tensor*, and rewrite the law of bone adaptation according with it. This second order tensor is "*is a symmetric second rank tensor that is a quantitative stereological measure of the microstructural arrangement of trabeculae and pores*" and the its principal directions coincide with the trabecular orientation. So, the elastic properties are function of volume fraction and the fabric tensor ***H*, *i.e.*, *E***=***E***(μ,***H***). The equilibrium equation correspond to the absence of trabecular alignment, when the trabeculae are aligned with the principal stress directions.

Bone adaptation – Fyhrie and Carter model

• In 1986 Fyhrie and Carter developed their bone adaptation model. They define the model as a *self optimization* model, and not as an evolutionary model.

• In some versions of the model bone is assumed isotropic with properties given by an exponential law:

$$E = E_0 \cdot (\rho)^p$$

where E is the Young modulus of bone for na apparent density ρ, E_0 is a parameter and p an exponent (E is given in MPa, ρ in g/cm^3, used values were E_0=3790 and p=3, from Carter and Hayes, 1977).

• When a multiload formulation is considered the evolution law is given by,

$$\rho = C\left[\sum_P n_P(\underline{\sigma}_P)^M\right]^{1/2M}$$

where ρ is the apparent density, n_P is the number of cycles for load P, $\underline{\sigma}_P$ is a scalar measure for stress and C and M are constants.

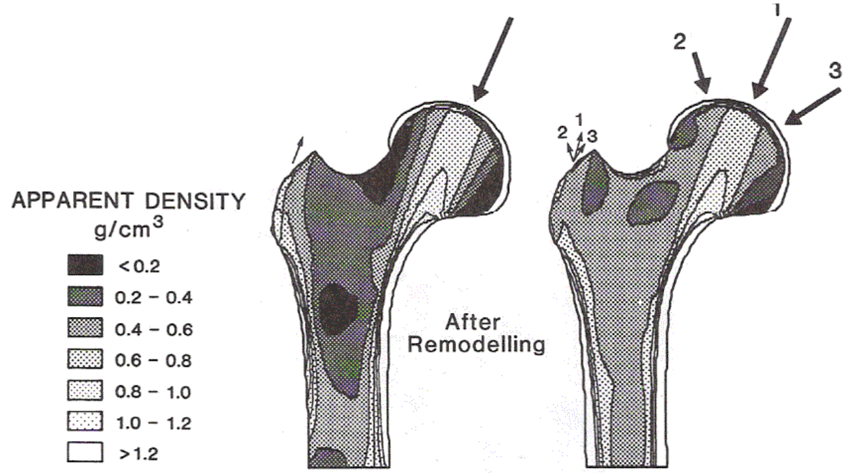


FIGURE 6.14. Results of a typical model of the kind used by Carter and co-workers. Less dense areas are indicated by *darker shading* as seen in scale at left. The model at *left* was loaded by a single set of hip and abductor forces *(arrows)*; that at *right* was loaded by three different loads (i.e., different *i* in Eq. 6.3), representing abduction, adduction, and normal hip positions. (Reproduced with permission from Carter et al., 1987b.)

Bone adaptation – Huiskes model

• In 1987, Huiskes and co-workers developed an evolutive model for bone adaptation based on the previous models and observations.

• The mechanical stimulus is the elastic strain energy density (it is a scalar value)

elastic strain energy density → $U=1/2.\sigma_{ij}\varepsilon_{ij}$

• Bone is considered an isotropic material

• The model addresses the problem of internal and external adaptation (indeed, the internal adaptation is more used in later works).

• The model assumes a plateau region, *i.e.* a range of strain energy the where the stimulus is null.

Bone adaptation – Huiskes model

- For internal bone adaptation the evolutive law can be written as,

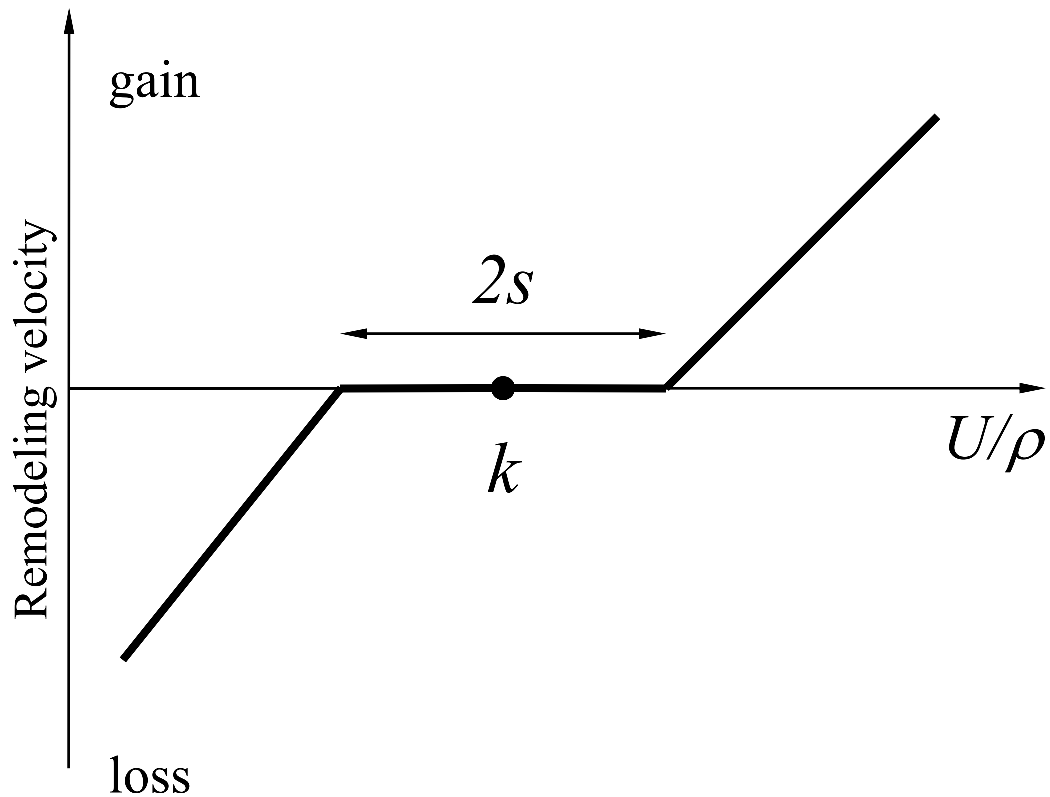
$$\frac{d\rho}{dt}=\begin{cases} B\left(\dfrac{U}{\rho}-(1-s)\cdot k\right), & \text{if } \dfrac{U}{\rho}<(1-s)\cdot k \\ 0, & otherwise \\ B\left(\dfrac{U}{\rho}-(1+s)\cdot k\right), & \text{if } \dfrac{U}{\rho}>(1+s)\cdot k \end{cases}$$

where:

- ρ is the apparent density
- t is the time variable ($d\rho/dt$ is the velocity of bone adaptation)
- U is the elastic strain energy density ($U=1/2.\sigma.\varepsilon$)
- k is a reference value
- B is a parameter
- s is a reference value to define the plateau (half of the plateau lenght).

Bone adaptation – Huiskes model

•the model can be graphically represented as follows:

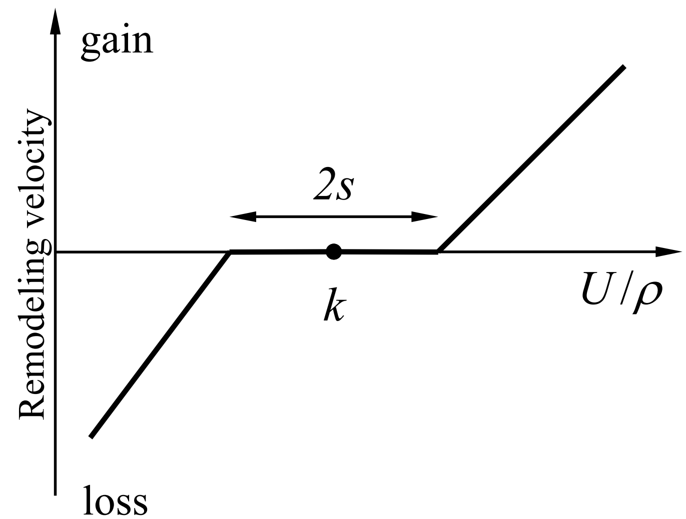


$$\frac{d\rho}{dt} = \begin{cases} B\left(\frac{U}{\rho} - (1-s)\cdot k\right), & \text{if } \frac{U}{\rho} < (1-s)\cdot k \\ 0, & otherwise \\ B\left(\frac{U}{\rho} - (1+s)\cdot k\right), & \text{if } \frac{U}{\rho} > (1+s)\cdot k \end{cases}$$

• the plateau represent a range of values where the stimulus is zero

Bone adaptation – Huiskes model

$$\frac{d\rho}{dt}=\begin{cases} B\left(\dfrac{U}{\rho}-(1-s)\cdot k\right), & \text{if } \dfrac{U}{\rho}<(1-s)\cdot k \\ 0, & otherwise \\ B\left(\dfrac{U}{\rho}-(1+s)\cdot k\right), & \text{if } \dfrac{U}{\rho}>(1+s)\cdot k \end{cases}$$



• in the pionner work (1987) the used value for s were s = 0%, 5%, 15%, 30%.

• in some works they used k = 0.0025 J/g (in the 1987 work, for external remodeling, they used for the periosteum surface a value of $U_0 = 5.03\times10^{-6}$ MPa).

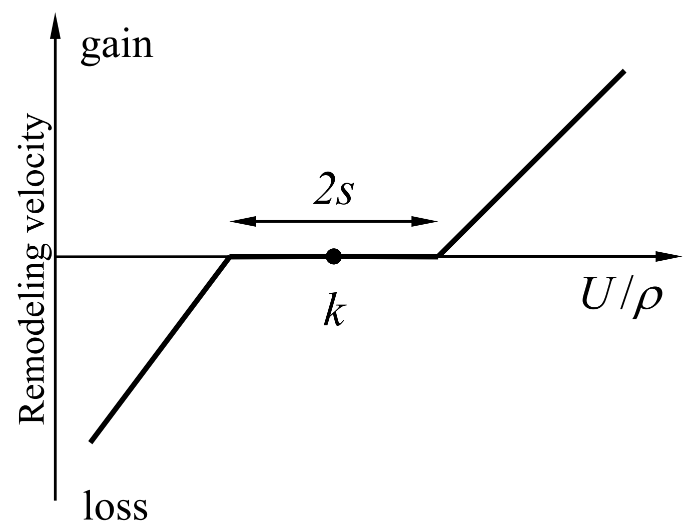
• the modulus of elasticity for bone can be defined as:

$$E = 3790.\rho^3$$

where E is given in Mpa and ρ in g/cm^3 for ρ = 0.01–1.74 g/cm^3

Bone adaptation – Huiskes model

$$\frac{d\rho}{dt} = \begin{cases} B\left(\frac{U}{\rho} - (1-s)\cdot k\right), & \text{if } \frac{U}{\rho} < (1-s)\cdot k \\ 0, & otherwise \\ B\left(\frac{U}{\rho} - (1+s)\cdot k\right), & \text{if } \frac{U}{\rho} > (1+s)\cdot k \end{cases}$$



• computationally the problem can be solved used a forward Euler method,

$$\frac{\partial\rho}{\partial t} = \mathcal{K} \tilde{\Rightarrow} \frac{\rho_{t+\Delta t} - \rho_t}{\Delta t} = \mathcal{K} \Rightarrow \rho_{t+\Delta t} = \rho_t + \Delta t \times \mathcal{K}$$

and thus

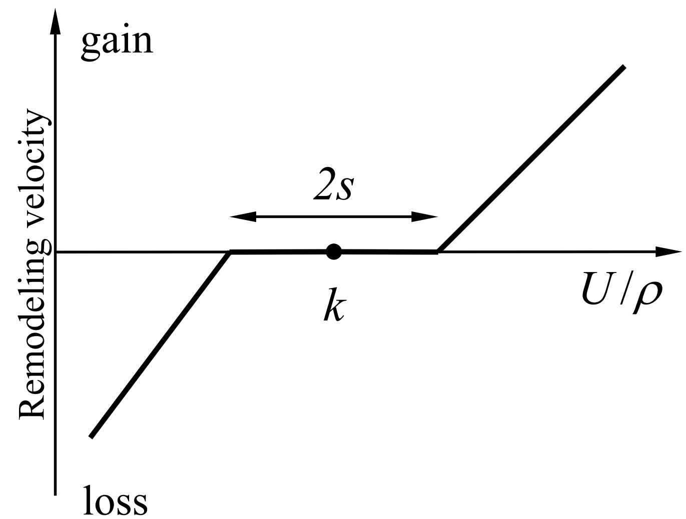
$$\rho_{t+\Delta t} = \begin{cases} \rho_t + \Delta t \times B\left(\frac{U}{\rho} - (1-s)\cdot k\right), & \text{if } \frac{U}{\rho} < (1-s)\cdot k \\ \rho_t, & otherwise \\ \rho_t + \Delta t \times B\left(\frac{U}{\rho} - (1+s)\cdot k\right), & \text{if } \frac{U}{\rho} > (1+s)\cdot k \end{cases}$$

• the value $\Delta t \times B$, is a step size and should be adjusted conveniently.

Bone adaptation – Huiskes model

- for multiple loads:

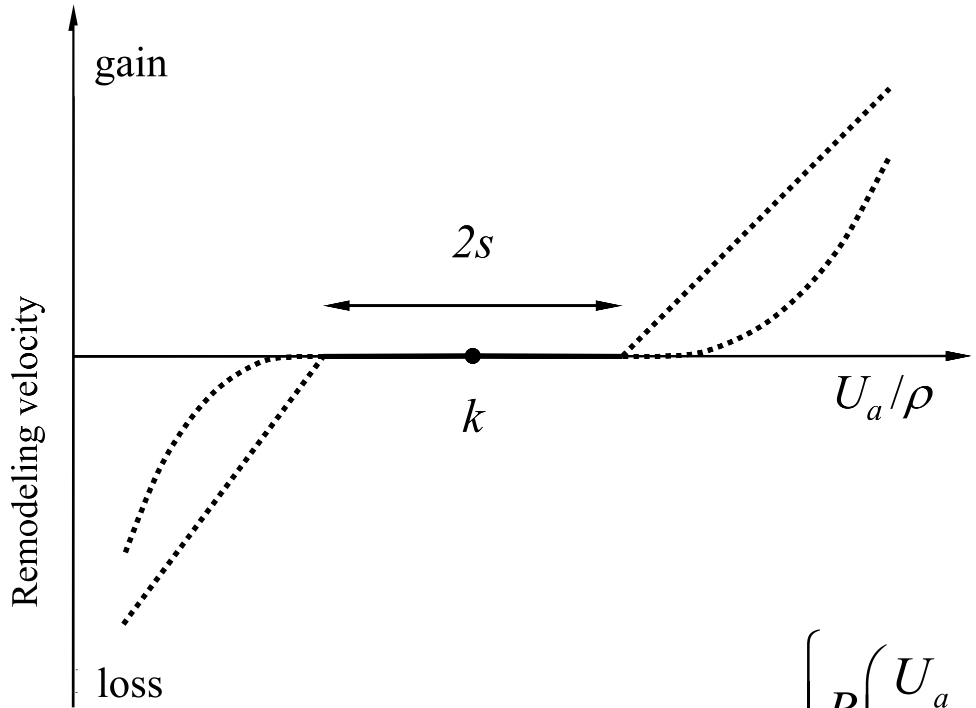
$$\frac{d\rho}{dt} = \begin{cases} B\left(\frac{U_a}{\rho} - (1-s)\cdot k\right), & \text{if } \frac{U_a}{\rho} < (1-s)\cdot k \\ 0, & otherwise \\ B\left(\frac{U_a}{\rho} - (1+s)\cdot k\right), & \text{if } \frac{U_a}{\rho} > (1+s)\cdot k \end{cases}$$



where U_a is na average of the strain energy, $\frac{U_a}{\rho} = \frac{1}{n}\sum_{i=1}^{n}\frac{U_i}{\rho}$ for n load cases.

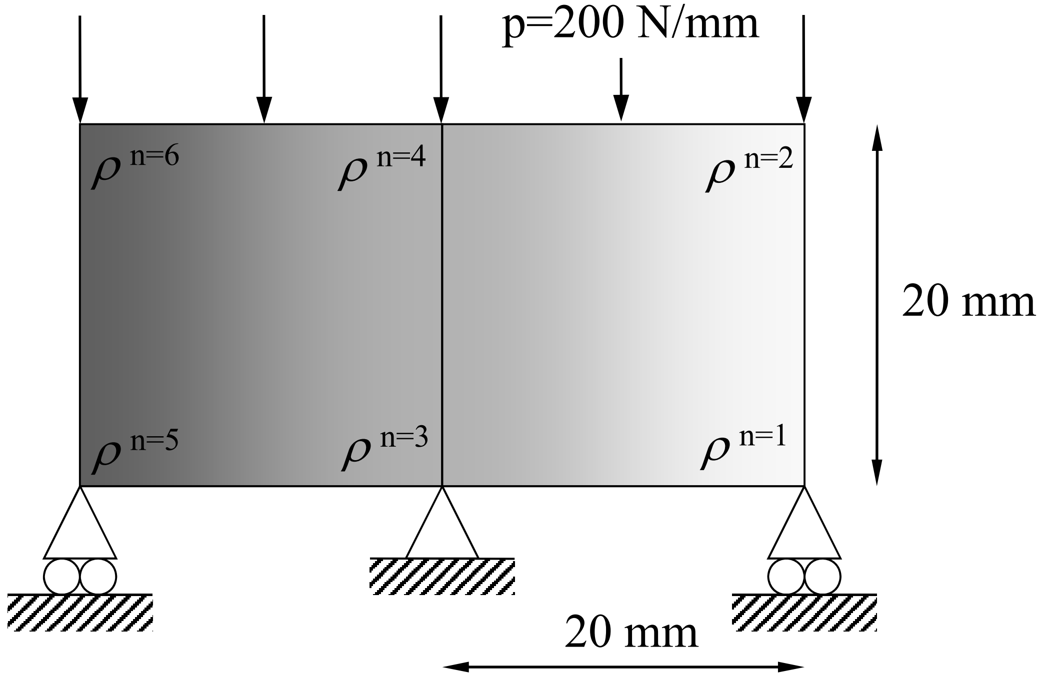
Bone adaptation – Huiskes model

• there are some variations of this law:



$$\frac{d\rho}{dt} = \begin{cases} B\left(\frac{U_a}{\rho} - (1-s)\cdot k\right)^p, & \text{if } \frac{U_a}{\rho} < (1-s)\cdot k \\ 0, & otherwise \\ B\left(\frac{U_a}{\rho} - (1+s)\cdot k\right)^p, & \text{if } \frac{U_a}{\rho} > (1+s)\cdot k \end{cases}$$

Huiskes model - example



Adaptation Law:

$$\frac{d\rho}{dt} = B\left(\frac{U}{\rho} - \cdot k\right) \Rightarrow$$

$$\Rightarrow \rho_{k+1} = \rho_k + \text{step} \times \left(\frac{U}{\rho} - k\right)$$

- used parameters: s = 0%, k = 0.0025 J/g = 2.5 N.mm/g
- material model: E= 3790.ρ^3, E em MPa, ρ = 0.01–1.74 g/cm^3
- **Initial densities: $\rho^{n=1,2}$ = 0.8 g/cm^3; $\rho^{n=3,4}$ = 1.2 g/cm^3 ; $\rho^{n=5,6}$ = 1.6 g/cm^3**

Huiskes model – example (iteration 1)

• Solving the linear elasticity problem (ABAQUS) →

$U^{n=1}$ =6.426 N/mm^2 ; $U^{n=2}$ = 6.366 N/mm^2

$U^{n=3}$ =2.887 N/mm^2 ; $U^{n=4}$ = 2.973 N/mm^2

$U^{n=5}$ =1.048 N/mm^2 ; $U^{n=6}$ = 1.077 N/mm^2

• Adaptation ($\rho_{k+1}=\rho_k+\text{step}\times(U/\rho - k)$):

nó 1: $U/\rho - k$ = 6.426/0.8–2.5 = 5.533; nó 2: $U/\rho - k$ = 6.366/0.8–2.5 = 5.458;

nó 3: $U/\rho - k$ = 2.887/1.2–2.5 = – 0.094; nó 4: $U/\rho - k$ = 2.973/1.2–2.5 = – 0.023;

nó 5: $U/\rho - k$ = 1.048/1.6–2.5 = – 1.845; nó 6: $U/\rho - k$ = 1.077/1.6–2.5 = – 1.827;

<u>With a step= 0.1 we obtain</u>:

nó 1: $\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$= 0.8+0.1×5.533 => $\rho^{n=1}$ =1.353 g/cm^3

nó 2: $\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$= 0.8+0.1×5.458 => $\rho^{n=2}$ =1.346 g/cm^3

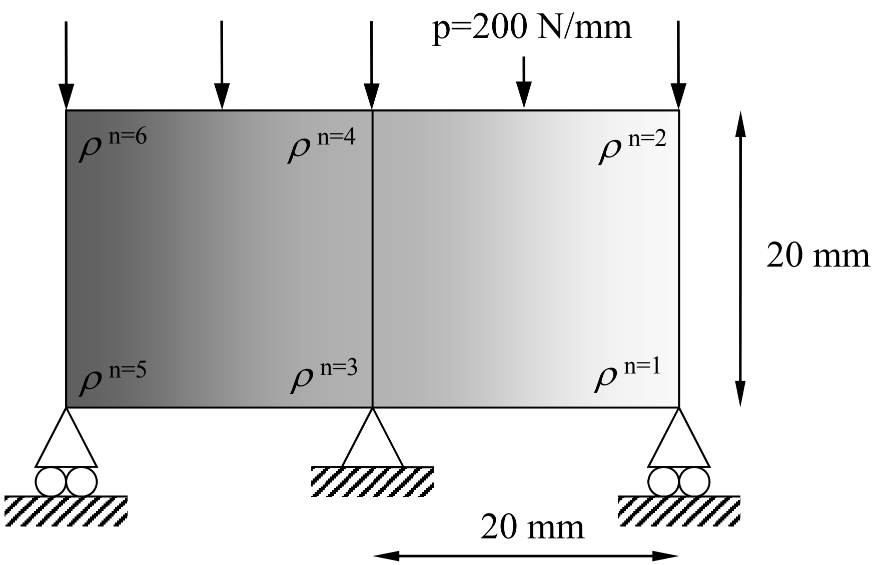
nó 3: $\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$= 1.2–0.1×0.094 => $\rho^{n=3}$ =1.191 g/cm^3

nó 4: $\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$= 1.2–0.1×0.023 => $\rho^{n=4}$ =1.198 g/cm^3

nó 5: $\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$= 1.6–0.1×1.845 => $\rho^{n=5}$ =1.416 g/cm^3

nó 6: $\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$= 1.6–0.1×1.827 => $\rho^{n=6}$ =1.417 g/cm^3

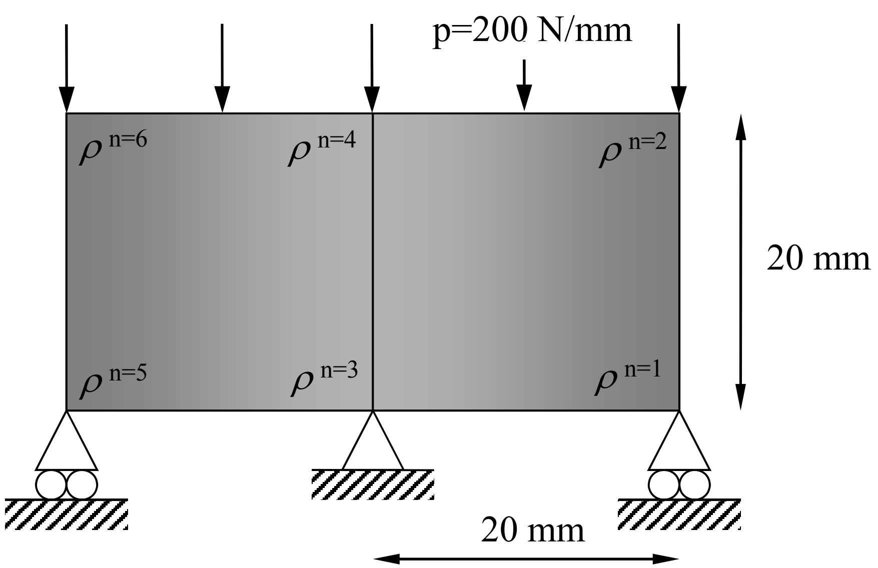
Huiskes model – example



$$\rho^{n=1,2} = 0.8\ \text{g/cm}^3$$

$$\rho^{n=3,4} = 1.2\ \text{g/cm}^3$$

$$\rho^{n=5,6} = 1.6\ \text{g/cm}^3$$



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$$\rho^{n=4} = 1.198\ \text{g/cm}^3$$
$$\rho^{n=5} = 1.416\ \text{g/cm}^3$$
$$\rho^{n=6} = 1.417\ \text{g/cm}^3$$

Huiskes model – example (iteration 2)

• Solving the linear elasticity problem(ABAQUS) →

$U^{n=1}$ =2.618 N/mm^2 ; $U^{n=2}$ = 2.618 N/mm^2

$U^{n=3}$ =2.459 N/mm^2 ; $U^{n=4}$ = 2.459 N/mm^2

$U^{n=5}$ =2.307 N/mm^2 ; $U^{n=6}$ = 2.037 N/mm^2

• Adaptation ($\rho_{k+1}=\rho_k$+passo×$(U/\rho - k)$):

nó 1: $U/\rho - k$ = 2.618/1.353–2.5 = – 0.565; nó 2: $U/\rho - k$ = 2.618/1.346–2.5 = – 0.555

nó 3: $U/\rho - k$ = 2.459/1.191–2.5 = – 0.435; nó 4: $U/\rho - k$ = 2.459/1.198–2.5 = – 0.447

nó 5: $U/\rho - k$ = 2.307/1.416–2.5 = – 0.871; nó 6: $U/\rho - k$ = 2.307/1.417–2.5 = – 0.872

With a step= 0.1 we obtain:

nó 1: $\rho_{k+1}=\rho_k$+passo×$(U/\rho - k)$= 1.353–0.1×0.565 => $\rho^{n=1}$ =1.297 g/cm^3

nó 2: $\rho_{k+1}=\rho_k$+passo×$(U/\rho - k)$= 1.346–0.1×0.555 => $\rho^{n=2}$ =1.291 g/cm^3

nó 3: $\rho_{k+1}=\rho_k$+passo×$(U/\rho - k)$= 1.191–0.1×0.435 => $\rho^{n=3}$ =1.148 g/cm^3

nó 4: $\rho_{k+1}=\rho_k$+passo×$(U/\rho - k)$= 1.198–0.1×0.447 => $\rho^{n=4}$ =1.155 g/cm^3

nó 5: $\rho_{k+1}=\rho_k$+passo×$(U/\rho - k)$= 1.416–0.1×0.871 => $\rho^{n=5}$ =1.329 g/cm^3

nó 6: $\rho_{k+1}=\rho_k$+passo×$(U/\rho - k)$= 1.417–0.1×0.872 => $\rho^{n=6}$ =1.330 g/cm^3

Huiskes model – example (iteration 3)

• Solving the linear elasticity problem(ABAQUS) →

$U^{n=1}$ =2.919 N/mm^2 ; $U^{n=2}$ = 2.919 N/mm^2

$U^{n=3}$ =2.813 N/mm^2 ; $U^{n=4}$ = 2.813 N/mm^2

$U^{n=5}$ =2.709 N/mm^2 ; $U^{n=6}$ = 2.709 N/mm^2

• Adaptation ($\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$):

nó 1: $U/\rho - k$ = 2.919/1.297–2.5 = – 0.249; nó 2: $U/\rho - k$ = 2.919/1.291–2.5 = – 0.239

nó 3: $U/\rho - k$ = 2.813/1.148–2.5 = – 0.050; nó 4: $U/\rho - k$ = 2.813/1.155–2.5 = – 0.065

nó 5: $U/\rho - k$ = 2.709/1.329–2.5 = – 0.462; nó 6: $U/\rho - k$ = 2.709/1.330–2.5 = – 0.463

<u>With a step= 0.1 we obtain</u>:

nó 1: $\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$= 1.297–0.1×0.249 => $\rho^{n=1}$ =1.272 g/cm^3

nó 2: $\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$= 1.291–0.1×0.239 => $\rho^{n=2}$ =1.267 g/cm^3

nó 3: $\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$= 1.148–0.1×0.050 => $\rho^{n=3}$ =1.143 g/cm^3

nó 4: $\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$= 1.155–0.1×0.065 => $\rho^{n=4}$ =1.149 g/cm^3

nó 5: $\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$= 1.329–0.1×0.462 => $\rho^{n=5}$ =1.283 g/cm^3

nó 6: $\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$= 1.330–0.1×0.463 => $\rho^{n=6}$ =1.284 g/cm^3

Huiskes model – example (iteration 4)

• Solving the linear elasticity problem(ABAQUS) →

$U^{n=1}$ =3.009 N/mm^2 ; $U^{n=2}$ = 3.009 N/mm^2

$U^{n=3}$ =2.963 N/mm^2 ; $U^{n=4}$ = 2.963 N/mm^2

$U^{n=5}$ =2.918 N/mm^2 ; $U^{n=6}$ = 2.918 N/mm^2

• Adaptation ($\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$):

nó 1: $U/\rho - k$ = 3.009/1.272–2.5 = – 0.134; nó 2: $U/\rho - k$ = 3.009/1.267–2.5 = – 0.125

nó 3: $U/\rho - k$ = 2.963/1.143–2.5 = 0.092; nó 4: $U/\rho - k$ = 2.963/1.149–2.5 = 0.079

nó 5: $U/\rho - k$ = 2.918/1.283–2.5 = – 0.226; nó 6: $U/\rho - k$ = 2.918/1.284–2.5 = – 0.227

<u>With a step= 0.1 we obtain </u>:

nó 1: $\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$= 1.272–0.1×0.134 => $\rho^{n=1}$ =1.259 g/cm^3

nó 2: $\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$= 1.267–0.1×0.125 => $\rho^{n=2}$ =1.255 g/cm^3

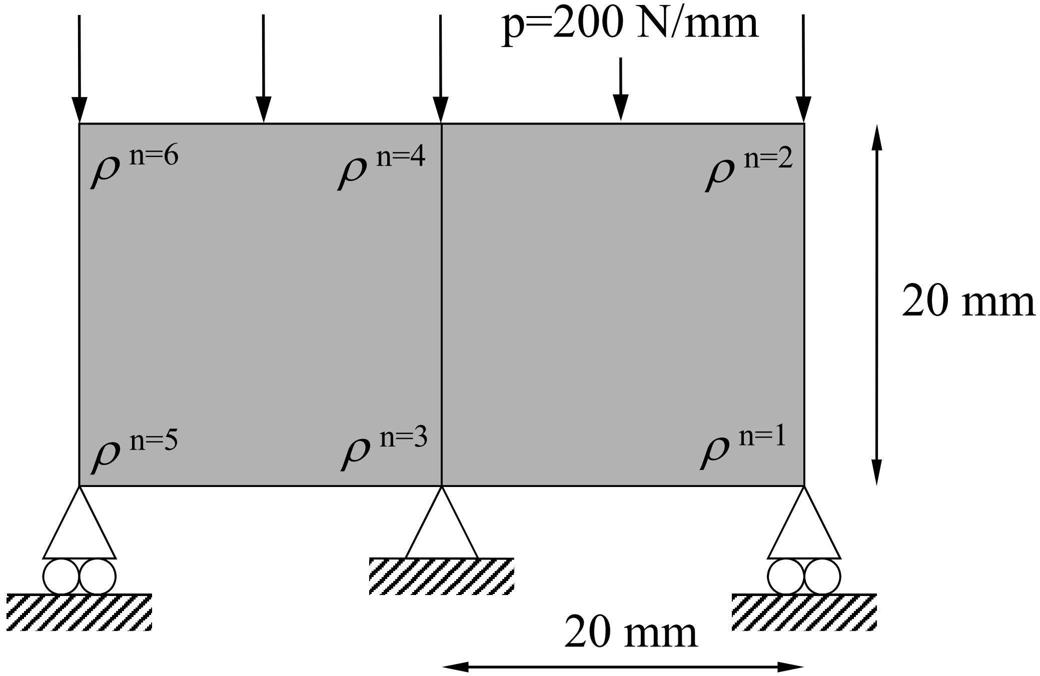
nó 3: $\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$= 1.143+0.1×0.092 => $\rho^{n=3}$ =1.152 g/cm^3

nó 4: $\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$= 1.149+0.1×0.079 => $\rho^{n=4}$ =1.157 g/cm^3

nó 5: $\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$= 1.283–0.1×0.226 => $\rho^{n=5}$ =1.260 g/cm^3

nó 6: $\rho_{k+1}=\rho_k+\text{passo}\times(U/\rho - k)$= 1.284–0.1×0.227 => $\rho^{n=6}$ =1.261 g/cm^3

Huiskes model – example



• The stationary solution, $d\rho/dt = 0$ is obtained for: $\boldsymbol{\rho^n \approx 1.2}$ **g/cm³**

Bone adaptation – Huiskes model

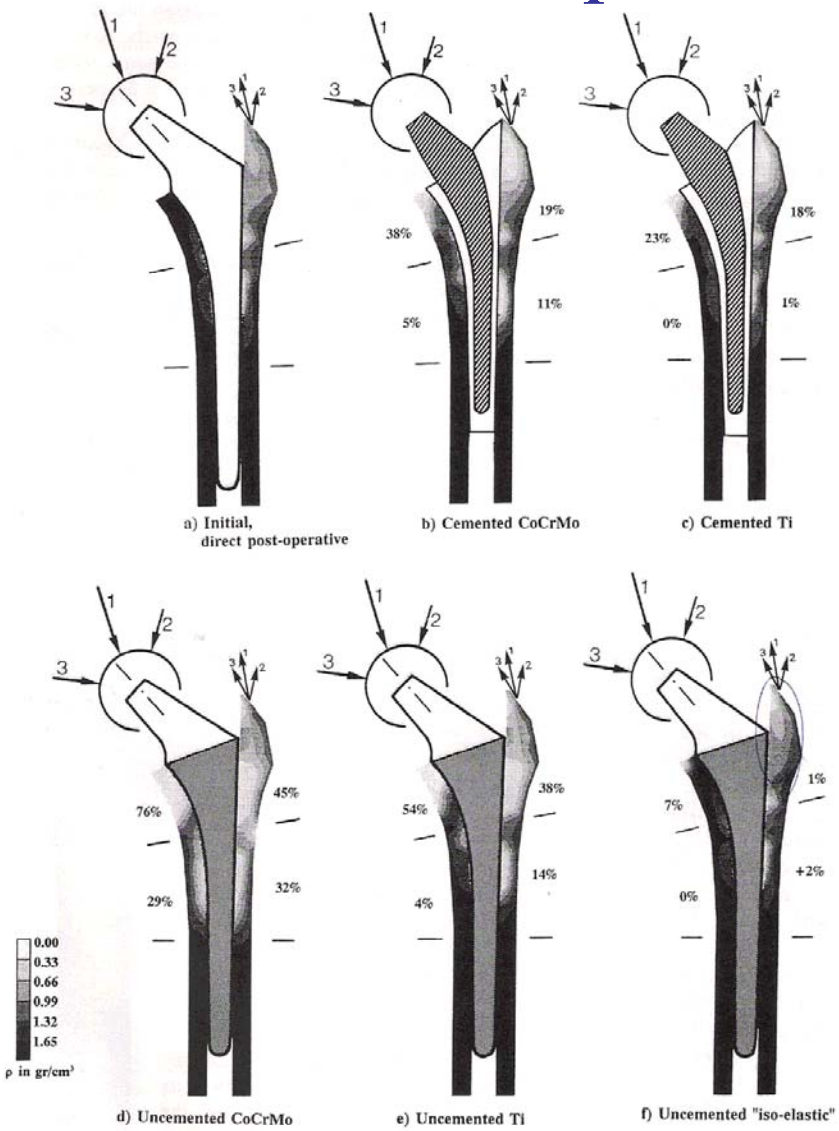


Fig. 6 Density distributions of the proximal femur with different stem types. The percentages bone loss in four different area's (Gruen's zones) are indicated.

- This model (for internal remodeling) was applied by Huiskes and co-workers, to study the bone adaptation, not only for an intact bone, but also to study the bone adaptation around implants. This way, it is possible to study the stress shield effect on the host bone.

Bone adaptation – model of Beaupré *et al.*

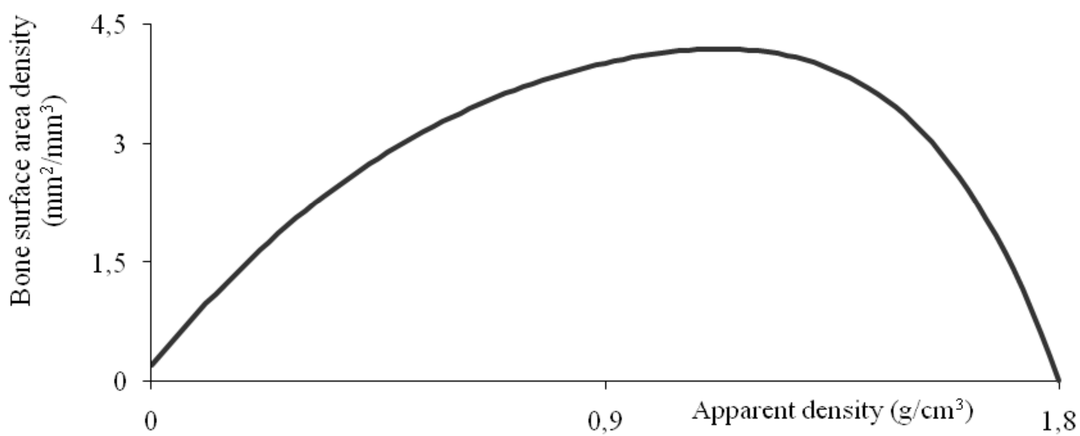
• In 1990, Beaupré, Orr and Carter, presented a theory for evolutionary bone adaptation based on the previous model but that also take in account the bone surface area.

• Bone is considered isotropic and properties obtained by na exponential law. The mechanical stimulus is the stress (similar to Fyhrie and Carter), and also consider multiple loads.

• The introduction of the bone surface area is an attempt to introduce information about the internal morphology of the trabecular bone. It is considered the trabecular surface play an important role on the internal bone remodeling. The potential for remodeling is related to the bone surface area.

$$\frac{d\rho}{dt} = A\left(\Psi \cdot \mu^2 - \Psi_0\right) \cdot S_v(\rho), \quad \Psi = \left[\sum_P n_P (\underline{\sigma}_P)^M\right]^{1/M}$$

where ρ represent the bone apparent density, t the time, μ the volume fraction, Ψ_0 is a reference value, S_v bone surface area density, n_P is the number of cycles for load P, $\underline{\sigma}_P$ is a scalar measure of stress and A, M are constants.



Bone adaptation – a brief summary of proposed models

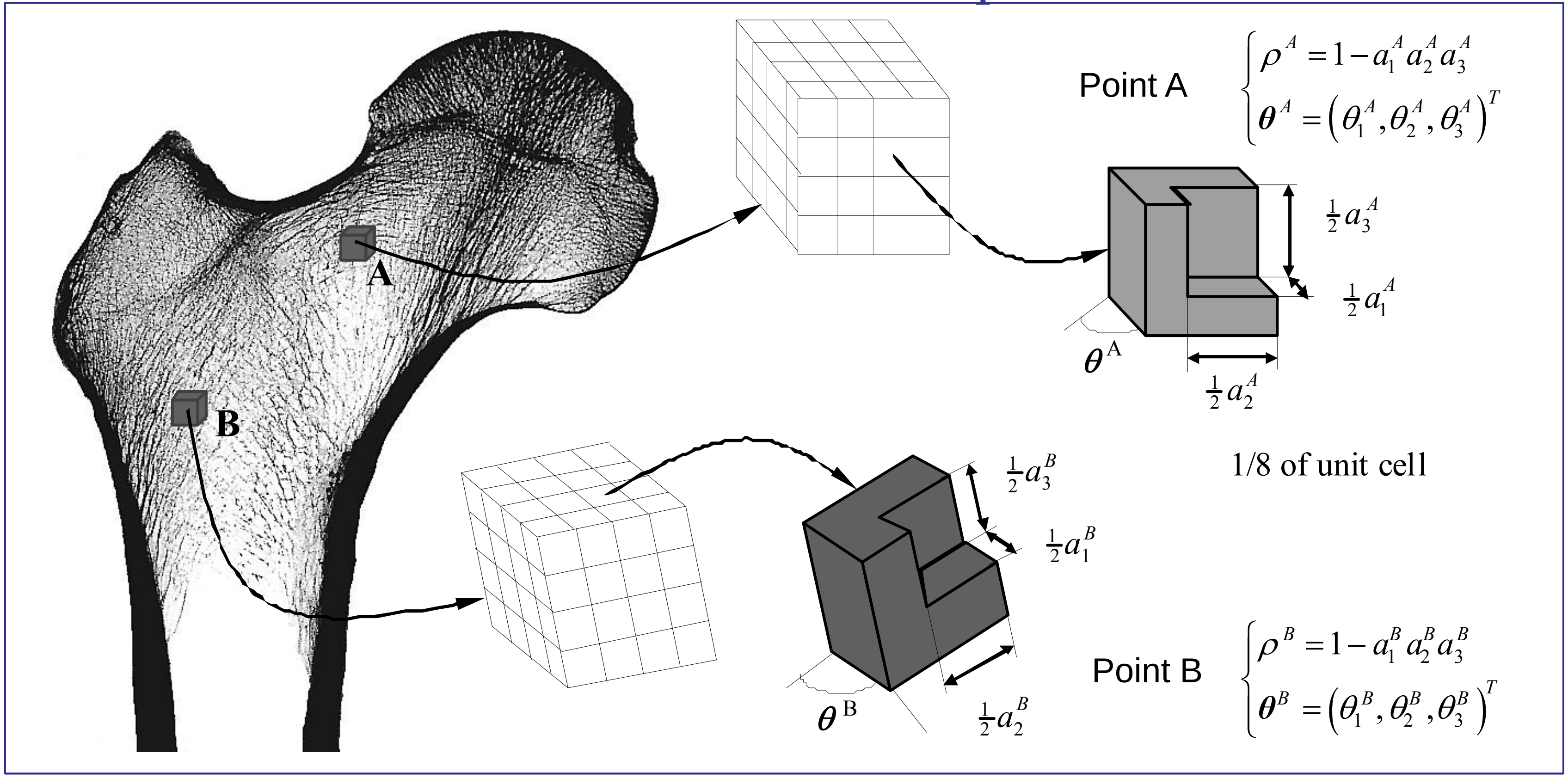
Considering bone as an isotropic structural material

- Hart *et al.,* A computational model for stress analysis of adaptive elastic materials with a view toward applications in strain-induced bone remodeling. J. Biomech. Eng., (1984)
- Huiskes *et al.*, 1987. Adaptive bone-remodeling theory applied to prosthetic-design analysis. J. Biomech. (1987)
- Carter *et al.*, Trabecular bone density and loading history: regulation of tissue biology by mechanical energy. J. Biomech. (1987)
- Beaupré *et al.*, An approach for time-dependent bone modeling and remodeling-theoretical development. J. Orth. Res. (1990)
- Weinans *et al.*, The behavior of adaptive bone-remodeling simulation models. J. Biomech. (1992)

Considering trabecular orientation or bone anisotropy

- Cowin *et al.,* An evolutionary Wolff's law for trabecular architecture, J. Biomech. Eng. (1992)
- Jacobs *et al.,* Adaptive bone remodeling incorporating simultaneous density and anisotropy considerations. J. Biomech. (1997)
- Hart *et al.,* Introduction to finite element based simulation of functional adaptation of cancellous bone. Forma (1997)
- Fernandes *et al.,* A model of bone adaptation using a global optimization criterion based on the trajectorial theory of Wolff. Comp. Meth. Biomech. Biomed. Eng. (1999)
- Rodrigues *et al.* Global and local material optimization applied to anisotropic bone adaptation. In. P. Perdersen and M.P. Bendsoe (Eds), Synthesis in Bio Solid Mechanics (1999)
- Doblaré and Garcia, Anisotropic bone remodelling model based on a continuum damage-repair theory. J. Biomech. (2002)
- P. Coelho, P. R. Fernandes, J.B. Cardoso, J. M. Guedes and H. C. Rodrigues, "Numerical Modeling of Bone Tissue Adaptation – A Hierarchical Approach for Bone Apparent Density and Trabecular Structure", Journal of Biomechanics, 42, pp. 830-837, 2009.

Bone adaptation – Lisbon model
a model based on structural optimization



P. Fernandes, H. Rodrigues and C. Jacobs, "A Model of Bone Adaptation Using a Global Optimisation Criterion Based on The Trajectorial Theory of Wolff", Computer Methods in Biomechanics and Biomedical Engineering, 2, pp. 125-138, **1999**

Bone adaptation – Lisbon model
a model based on structural optimization

$$\min_{\boldsymbol{a},\boldsymbol{\theta}} \left[\sum_{P=1}^{NC} \alpha^P \left(\int_{\Gamma_f} f_i^P u_i^P \, d\Gamma \right) + \kappa \int_{\Omega_b} \rho(\boldsymbol{a}) \, d\Omega \right]$$

compliance (maximize stiffness)

Total bone mass

κ – metabolic cost associated to bone apposition

s.t.

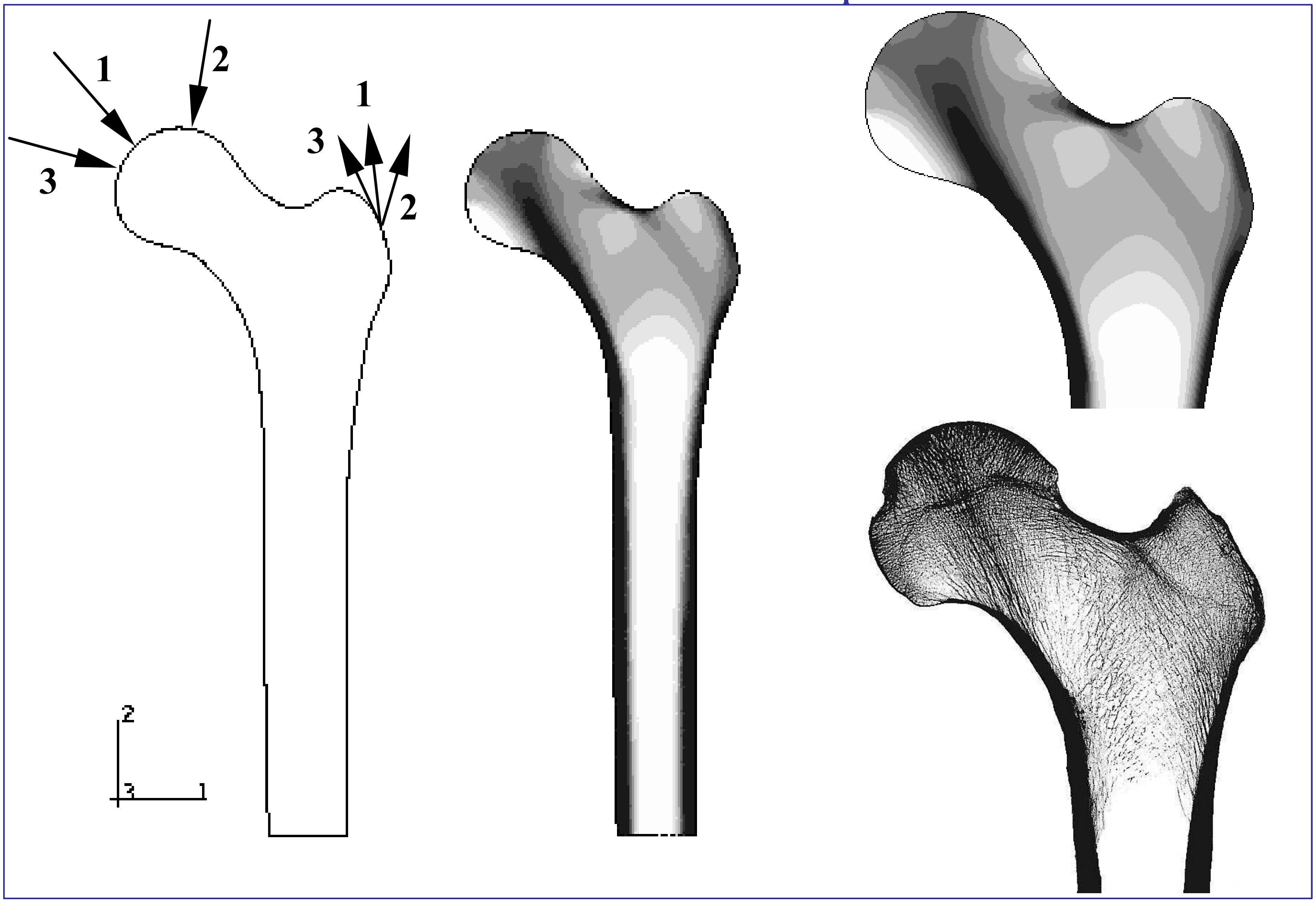
$$0 \le a_i \le 1 \quad , \quad i = 1,2,3$$ and *the equilibrium equation*

Law of Bone Remodeling:

$$\sum_{P=1}^{NC} \alpha^P \left[\frac{\partial E_{ijkl}^e}{\partial \boldsymbol{a}^e} e_{kl}\left(\boldsymbol{u}^P\right) e_{ij}\left(\boldsymbol{u}^P\right) \right] - \kappa \cdot \frac{\partial \rho^e}{\partial \boldsymbol{a}^e} = 0$$

It is the stationarity condition of the above problem and it represents the law of bone remodelling in the sense that whenever it holds the remodelling equilibrium is achieved.

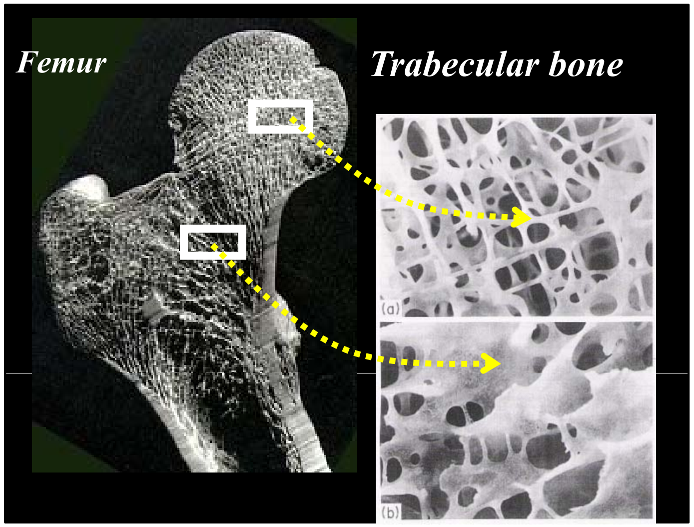
Bone adaptation – Lisbon model
a model based on structural optimization



Bone adaptation – Lisbon model
a model based on structural optimization
<u>multi-scale model</u>

- Bone is a hierarchical (multi-scale) material: several organizational levels can be identified from the macroscale to the nanoscale.

- The two top levels correspond to the entire bone and trabecular structure (density, mech. properties, material symmetry).



P. Coelho, P. R. Fernandes, J.B. Cardoso, J. M. Guedes and H. C. Rodrigues, “Numerical Modeling of Bone Tissue Adaptation – A Hierarchical Approach for Bone Apparent Density and Trabecular Structure”, Journal of Biomechanics, 42, pp. 830-837, **2009.**

P.G. Coelho, P.R. Fernandes, H.C. Rodrigues, “Multiscale Modeling of Bone Tissue with Surface and Permeability Control”, Journal of Biomechanics, 44(2), pp.321-329, **2011.**

Bone adaptation – Lisbon model - **multi-scale model**

Macro scale

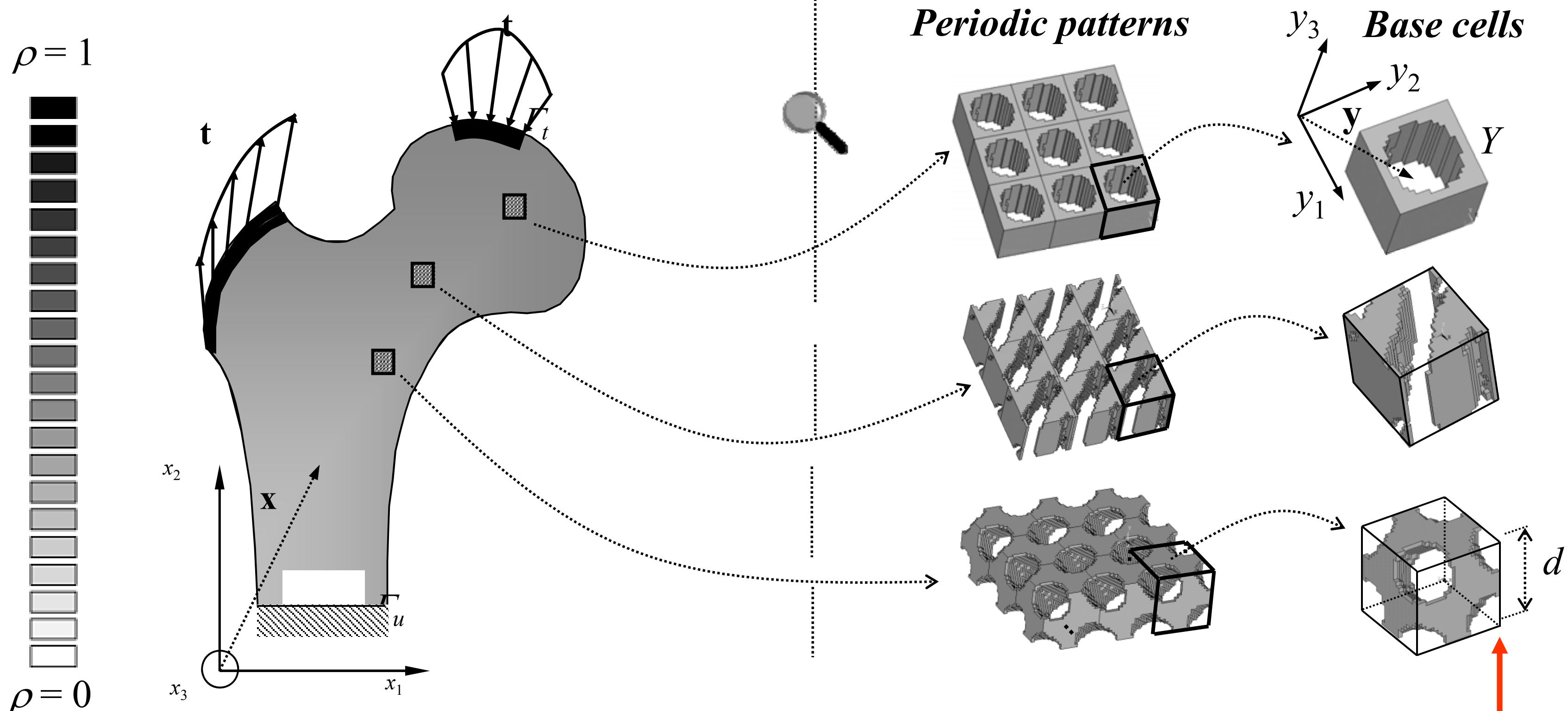
(Bone apparent density)

Microscale

(Trabecular architecture)

Macro-density field $\rho(\mathbf{x}) \in]0,1]$

Local or micro-density field $\mu(\mathbf{y}) \in]0,1]$



Homogenization ← *Interpolations schemes* → **SIMP**

Biological factors

Two material distribution problems – Global/Local

Bone adaptation – Lisbon model - **multi-scale model**

Law of Bone remodeling

In the multiscale model the apparent density ρ depends on a micro field μ which defines the trabecular architecture (microstructure):

$$\rho(\mathbf{x}) = \frac{1}{|Y|}\int_Y \mu(\mathbf{x},\mathbf{y})\,\mathrm{d}Y, \quad \forall \mathbf{x} \in \Omega$$

The value of μ is defined by the evolution law:

$$\frac{\mathrm{d}\mu}{\mathrm{d}t} = \frac{\partial E^{\mathrm{H}}_{ijkl}(\mu)}{\partial \mu(\mathbf{x},\mathbf{y})}\sum_{r=1}^{P}\left[\alpha^r \varepsilon_{ij}(\mathbf{u}^r)\,\varepsilon_{kl}(\mathbf{u}^r)\right] - k$$

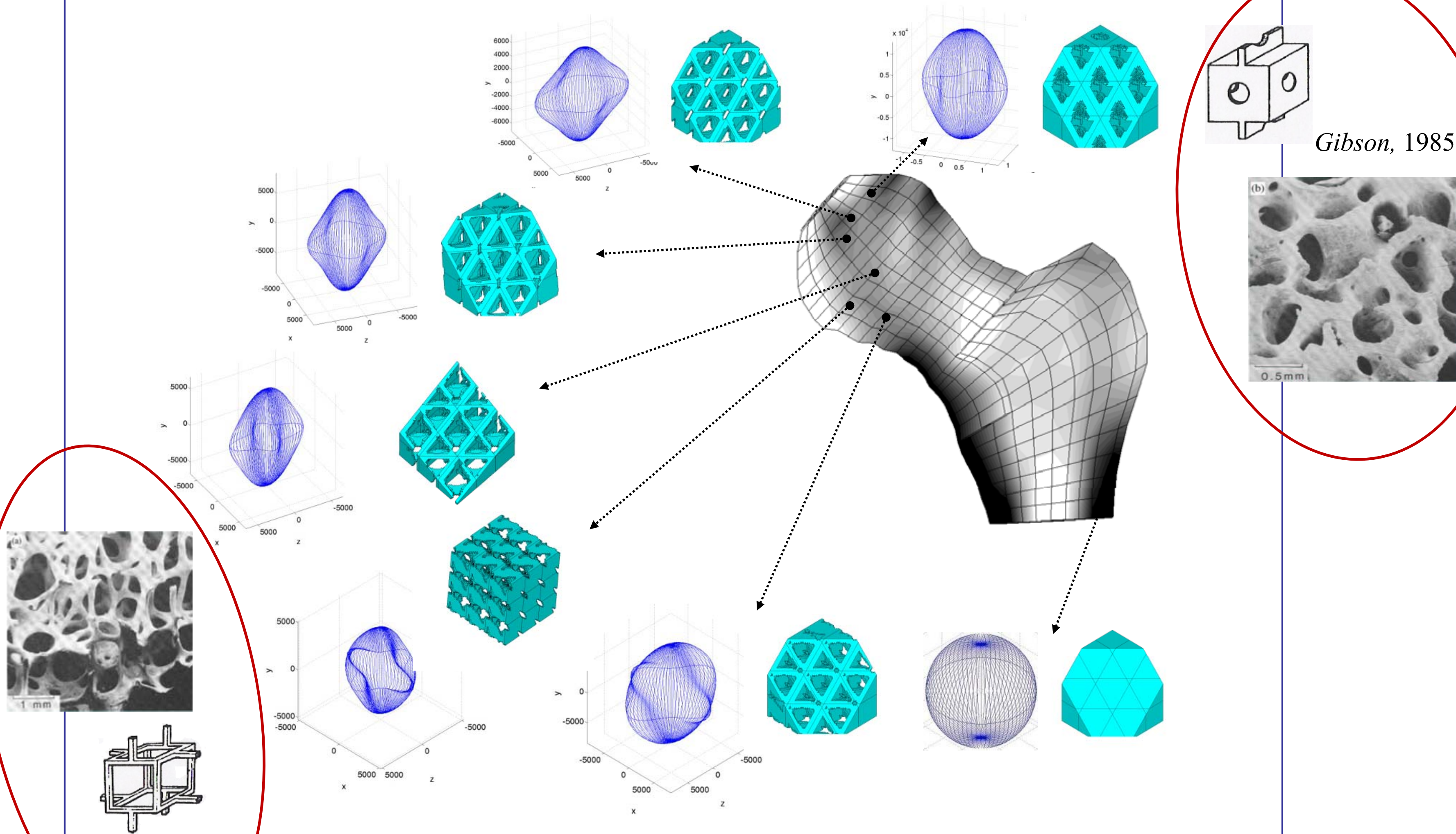
Cost of bone formation (k)

Mechanical Stimulus (the sum term)

This equation correspond to the law of bone remodeling in the sense that when $\mathrm{d}\mu / \mathrm{d}t = 0$, the remodelling equilibrium is achieved.

Bone adaptation – Lisbon model - **multi-scale model**

Results –bone anisotropy

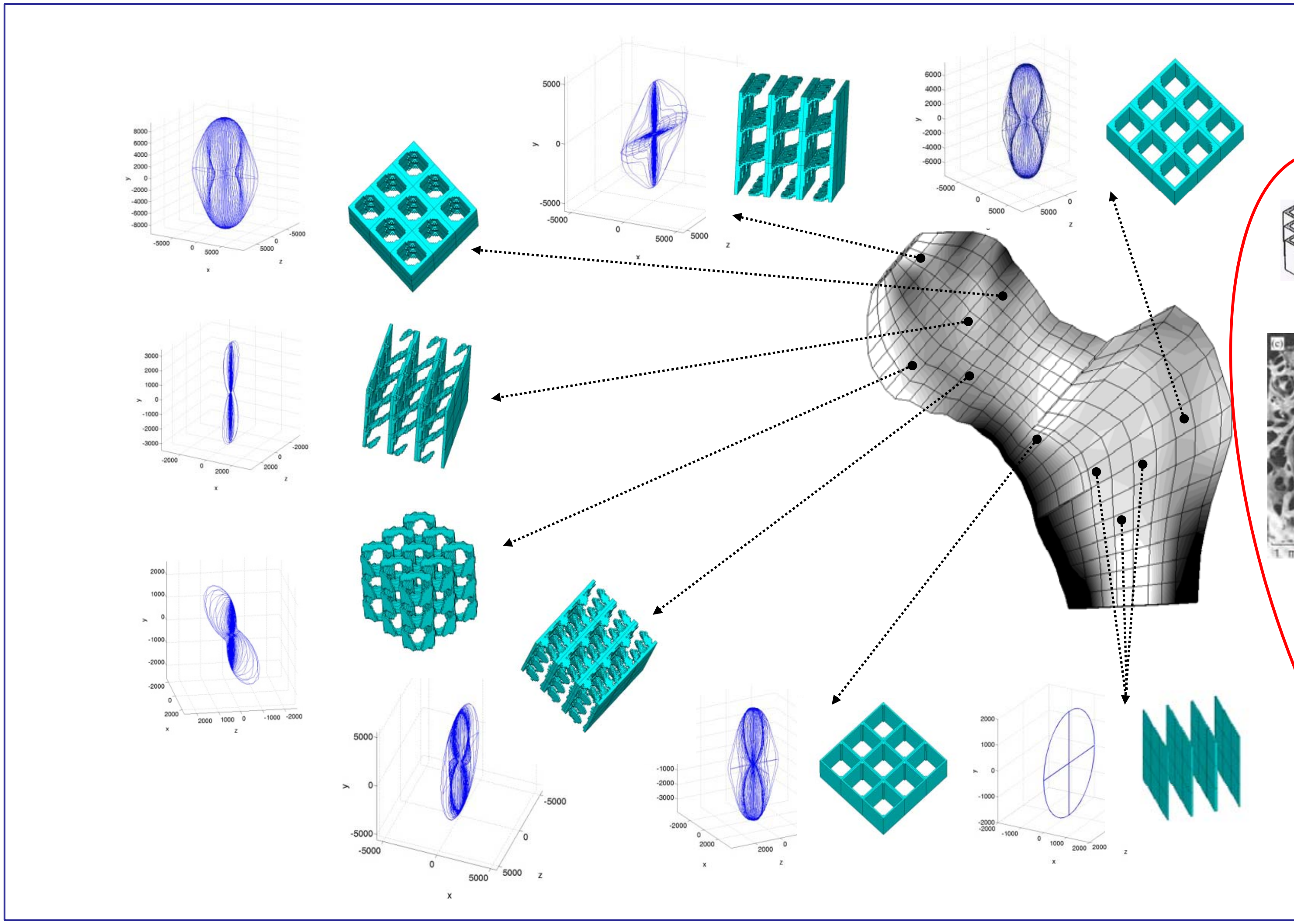


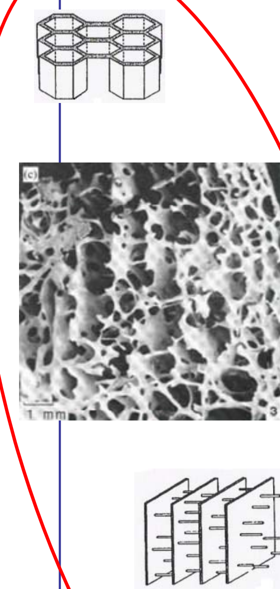
Gibson, 1985

Gibson, 1985

Bone adaptation – Lisbon model - **multi-scale model**

Results –bone anisotropy

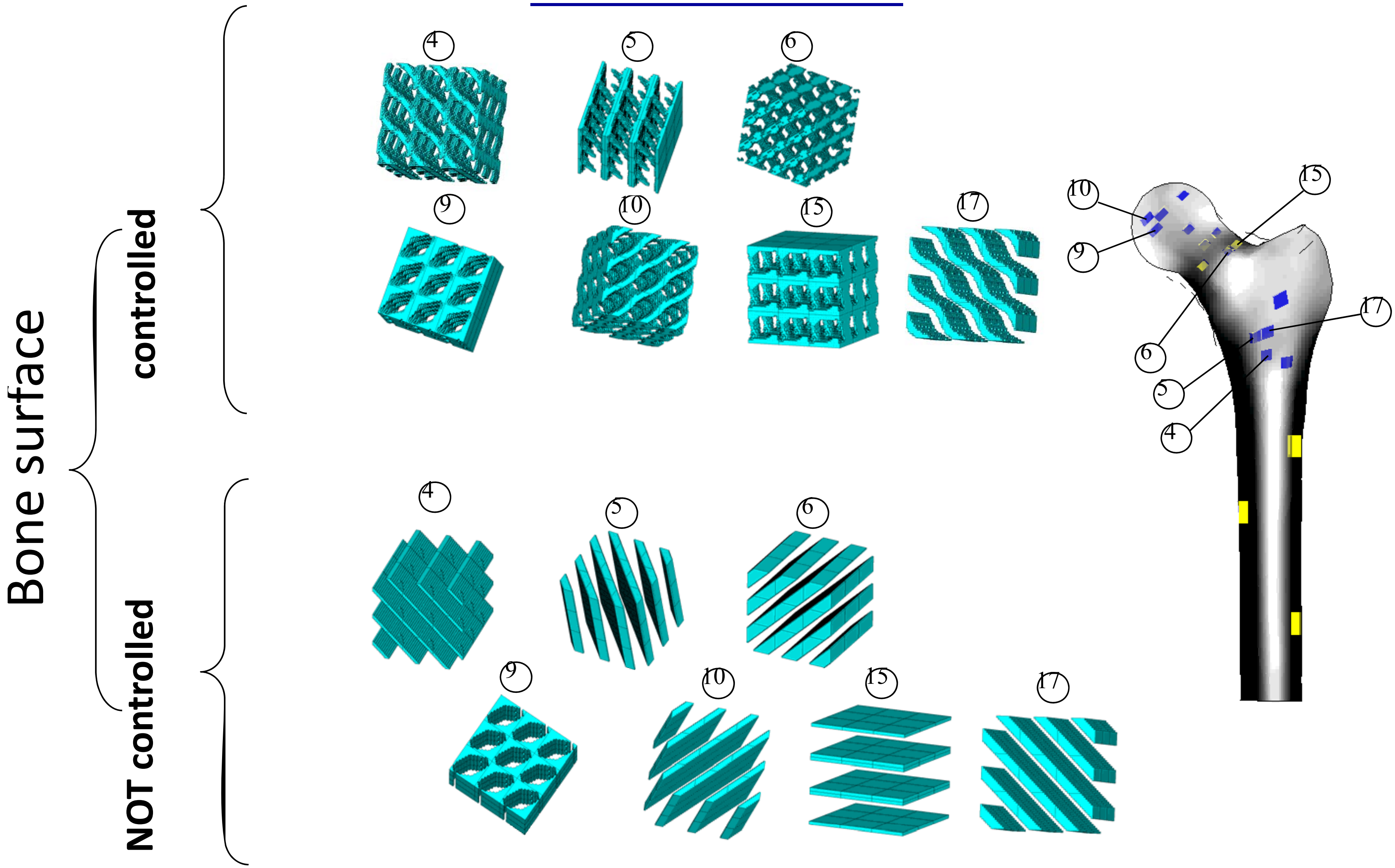




Gibson, 1985

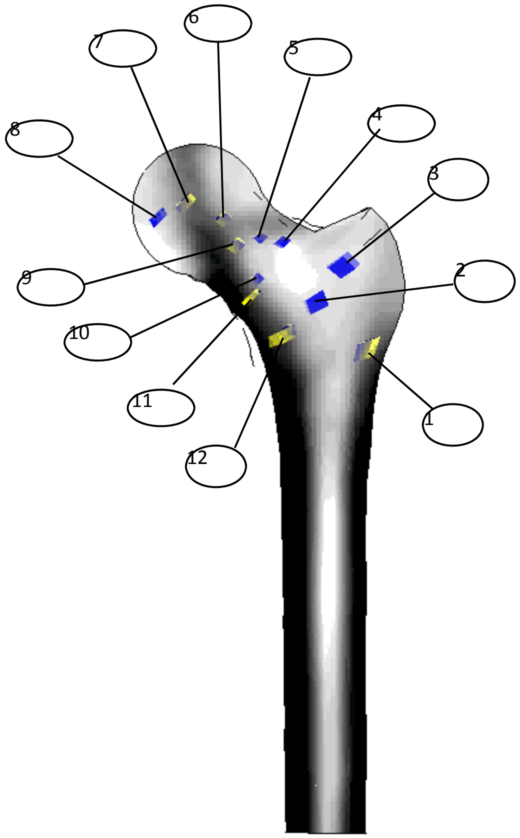
Bone adaptation – Lisbon model - **multi-scale model**

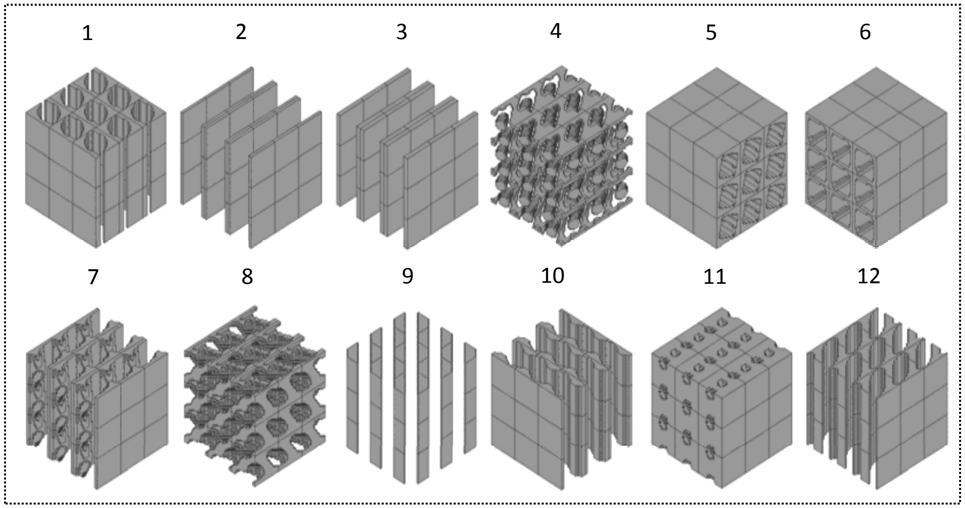
Bone surface control

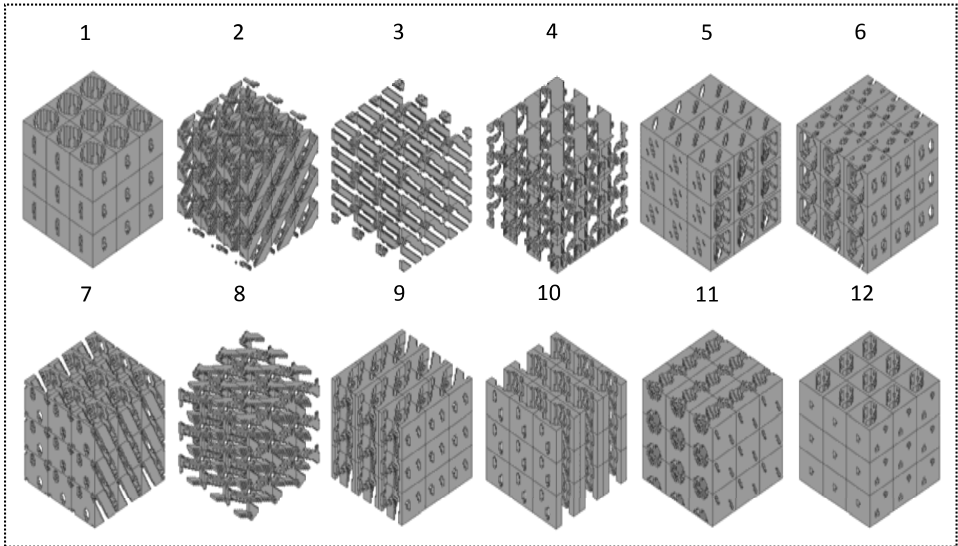


Bone adaptation – Lisbon model - **multi-scale model**

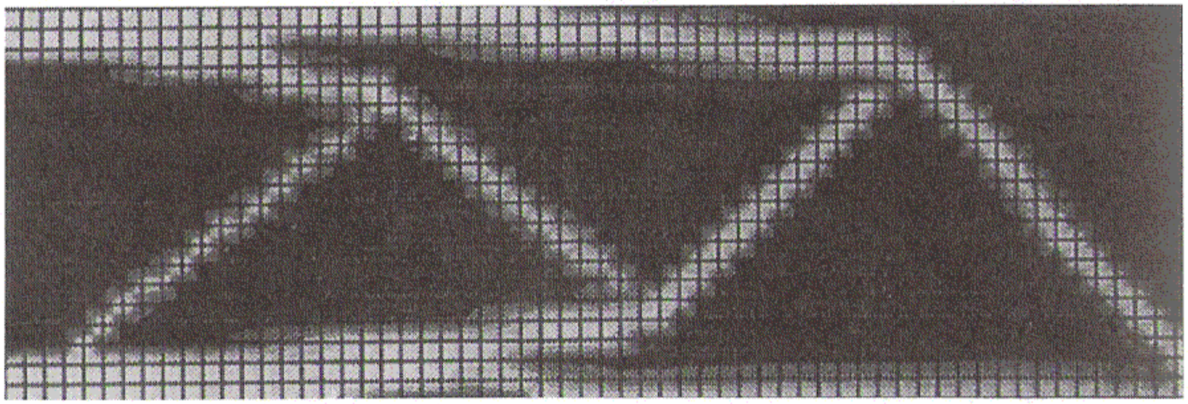
permeability control







Bone adaptation – comparison with structural optimization problems



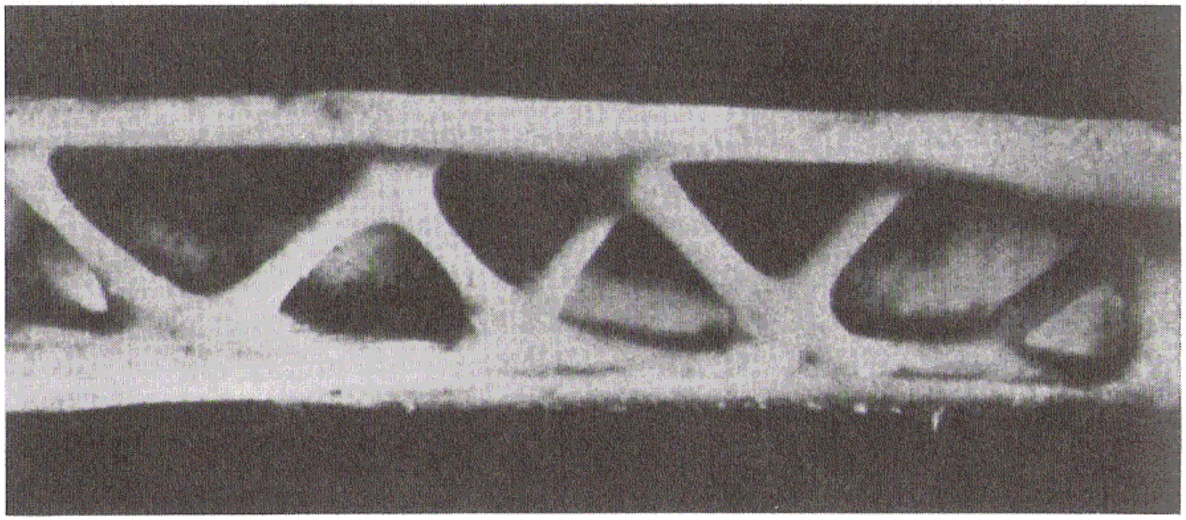


FIGURE 6.17. *Top:* Optimized design of an "aircraft support beam." (Reprinted from P. Pedersen, Ed., *Optimal Design with Advanced Materials,* 1993, p. 31, with kind permission from Elsevier Science–NL, Sara Burgerhartstraat 25, 1055 KV Amsterdam, The Netherlands.) *Bottom:* Photograph of trabecular struts within a vulture's wing. (Reproduced with permission from Thompson, 1942.)

Bone adaptation – numerical issues
checkerboard

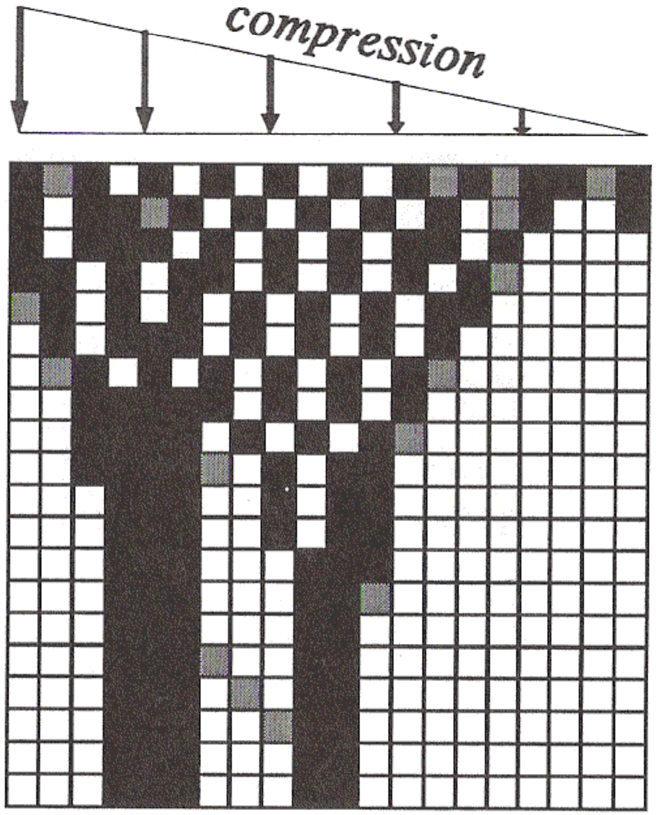


FIGURE 6.15. An example of an early self-trabeculating finite element model. A compressive load was applied to the upper surface, diminishing from left to right. The density of the material in each element is represented by a gray scale. Most elements have either maximum *(black)* or minimum *(white)* density. Two walls of cortical bone have developed below a metaphyseal region. Note the "checkerboarding" effect described in the text. (Reproduced from *J Biomechanics,* Vol. 25, Weinans et al., The behavior of adaptive bone-remodeling simulation models, 1425–1441, 1992, with kind permission from Elsevier Science Ltd., The Boulevard, Langford Lane, Kidlington 0X5 IGB, UK.)

• the numerial solution is sometimes instable and can lead to a final solution with checkerboard patterns.

• Usually, it appears in a final stage of the iterative process where intermediate densities should appear.

Bone adaptation – numerical issues
checkerboard

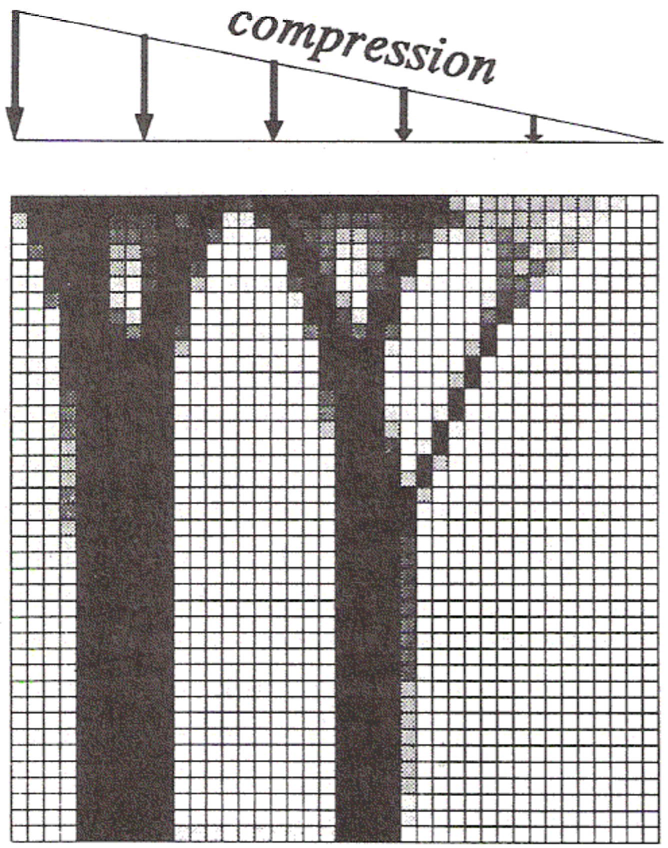
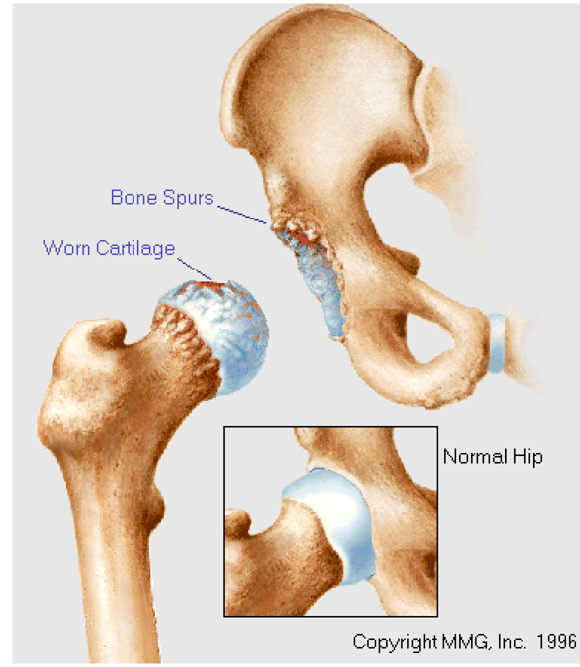


FIGURE 6.16. Self-trabeculating model similar to that of Fig. 6.15 but with a finer mesh. Checkerboarding has been eliminated by using Eq. 6.8. (Reproduced from *J Biomechanics*, Vol. 27, Mullender et al., 1389–1394, 1994, with kind permission from Elsevier Science Ltd., The Boulevard, Langford Lane, Kidlington 0X5 IGB, UK.)

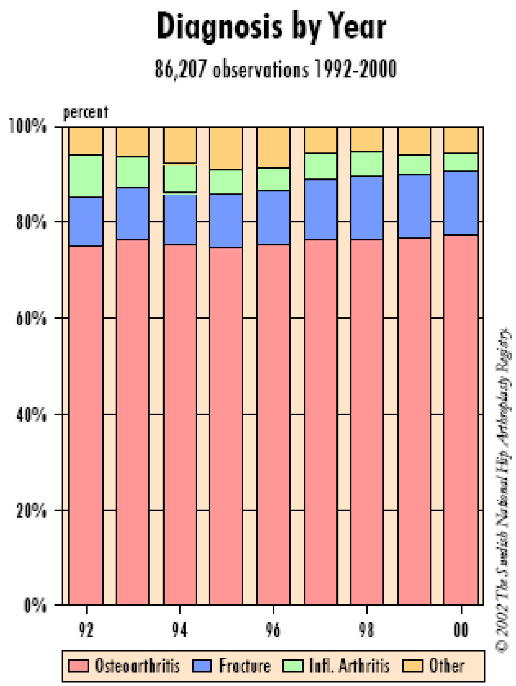
- a possible solution is to assume that the stimulus at a point is an average value of the stimulus at the neighbor points.

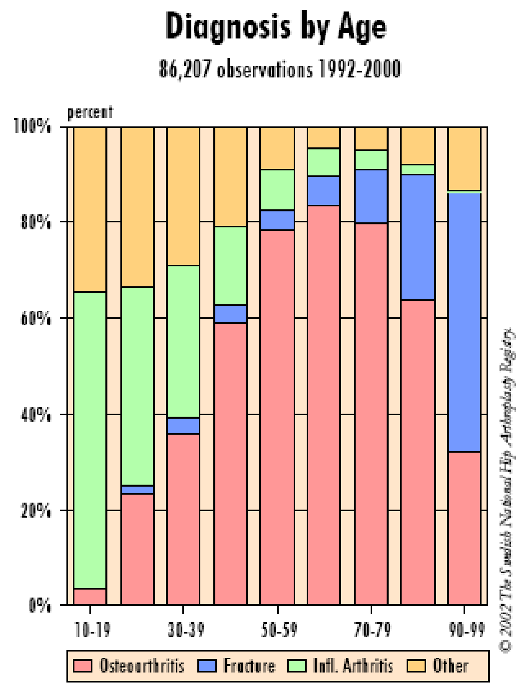
Orthopedic implants – articular joint diseases

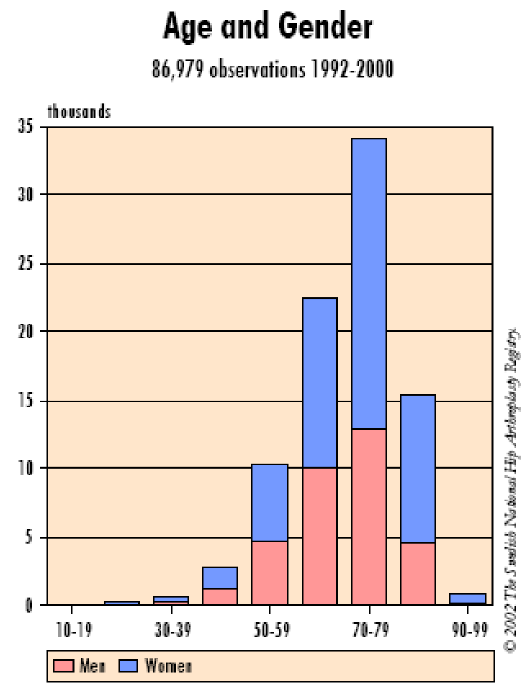


- osteoarthritis is the more usually cause of joint pain
- the substitution of the natural joint by an artificial one is a solution for these problems.

Orthopedic implants

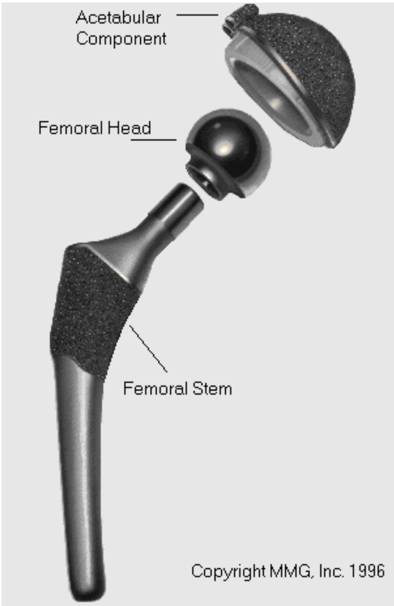


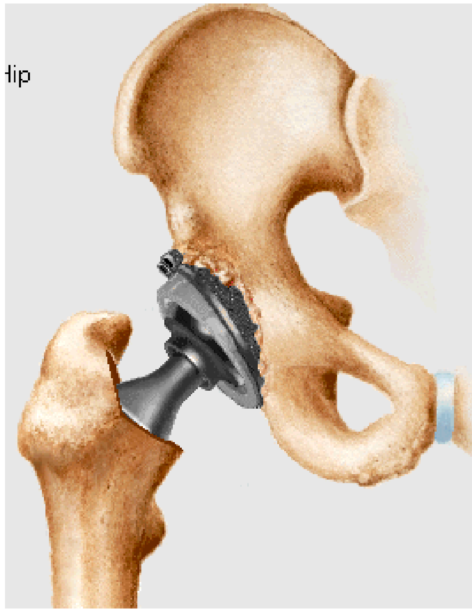


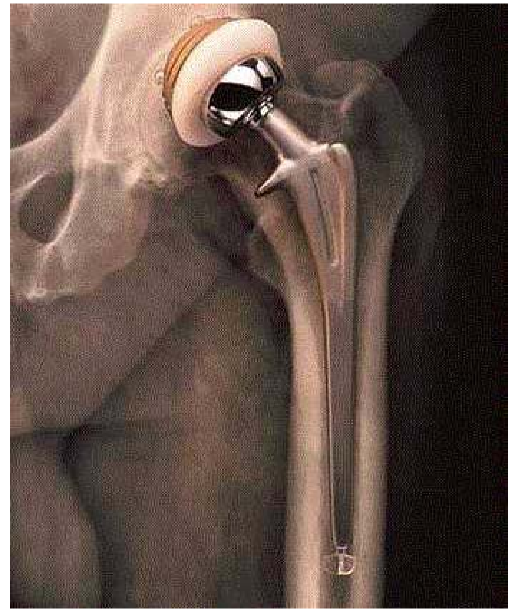


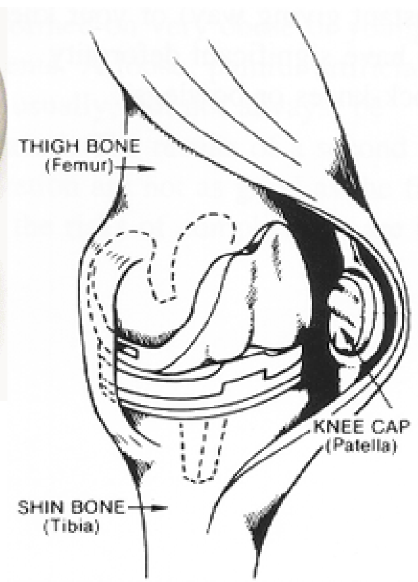
- 0.5 to 1 million of total hip arthroplasty per year worldwide.
- the principal causes are the osteoarthritis, rheumatoid arthritis, osteonecrosis and fracture.
- the biggest cause is osteoarthritis.

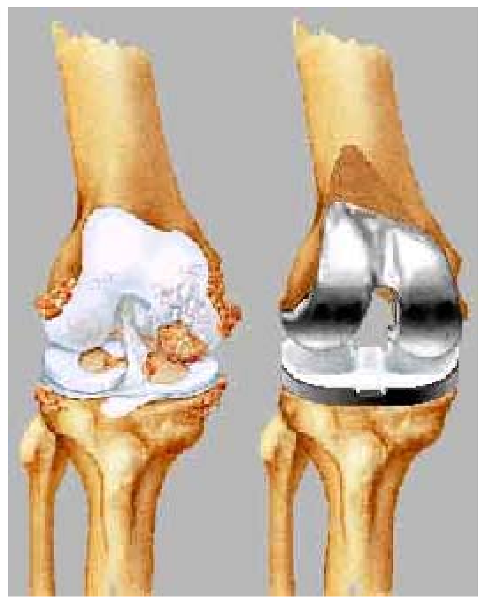
Orthopedic implants – total joint replacement

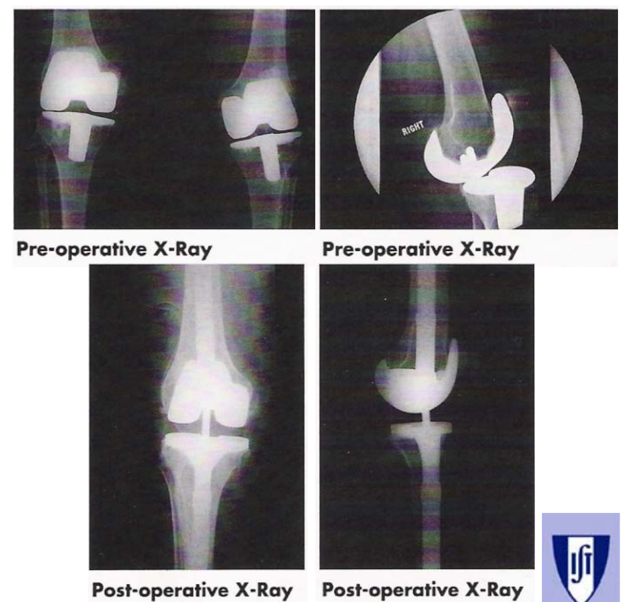






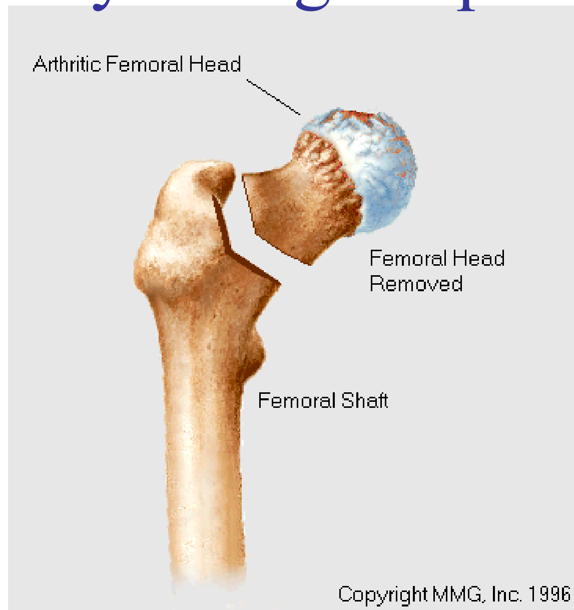




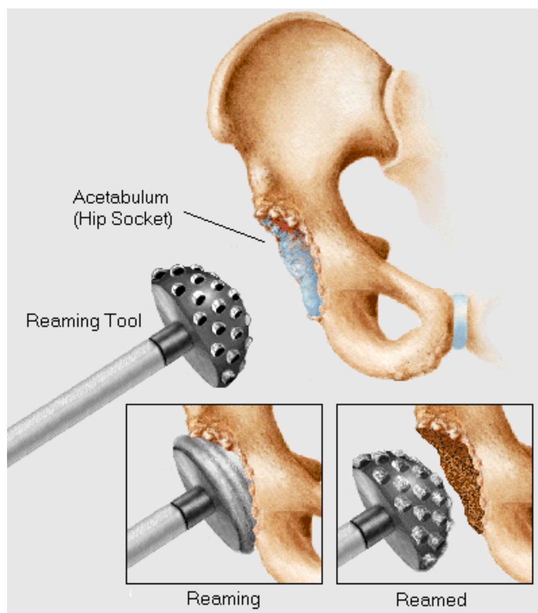


total hip arthroplasty – surgical procedure

Femoral head removed

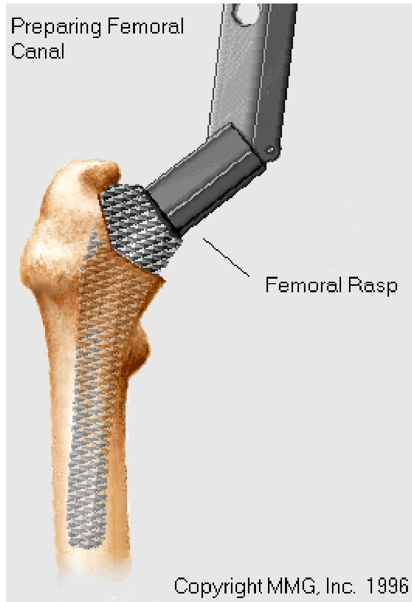


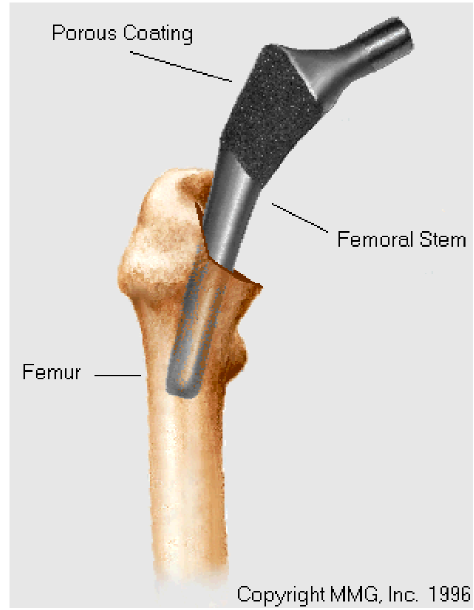
Acetabular component

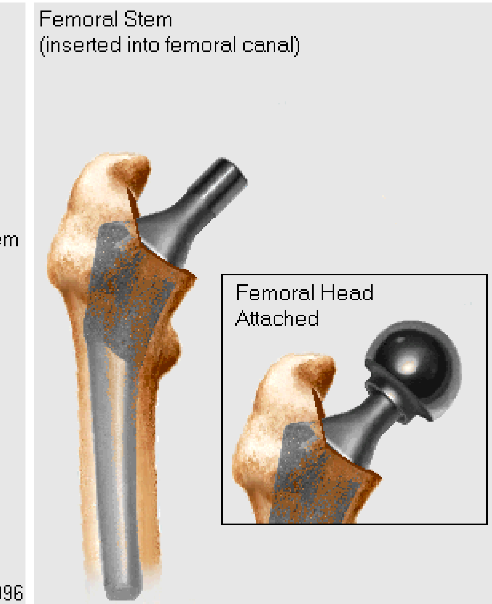


total hip arthroplasty – surgical procedure

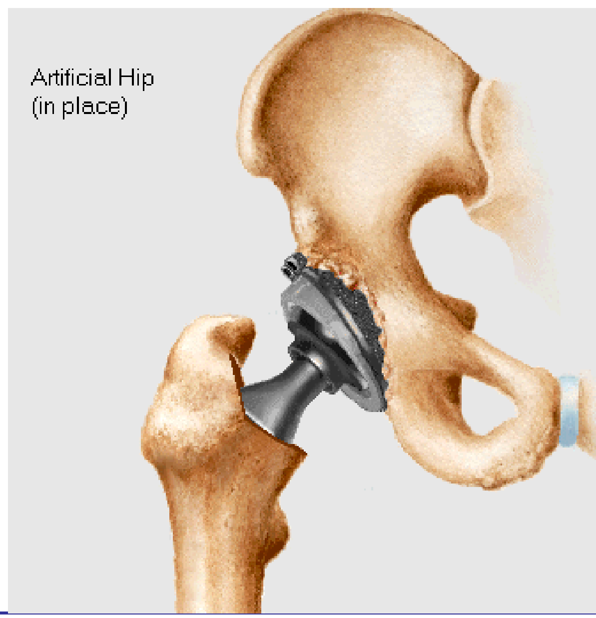
Femoral component (stem) – in general is made of Co-Cr, titanium or steel, the head is usually on Co-Cr or ceramic.

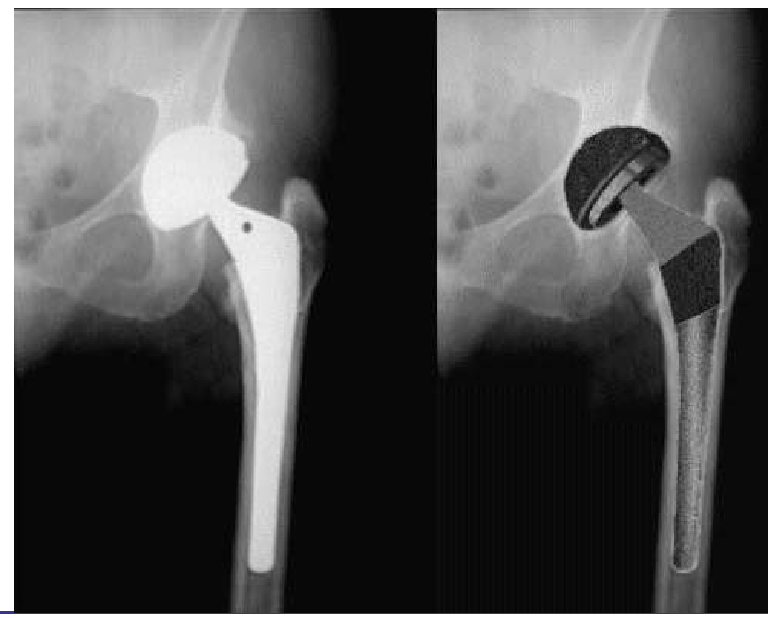






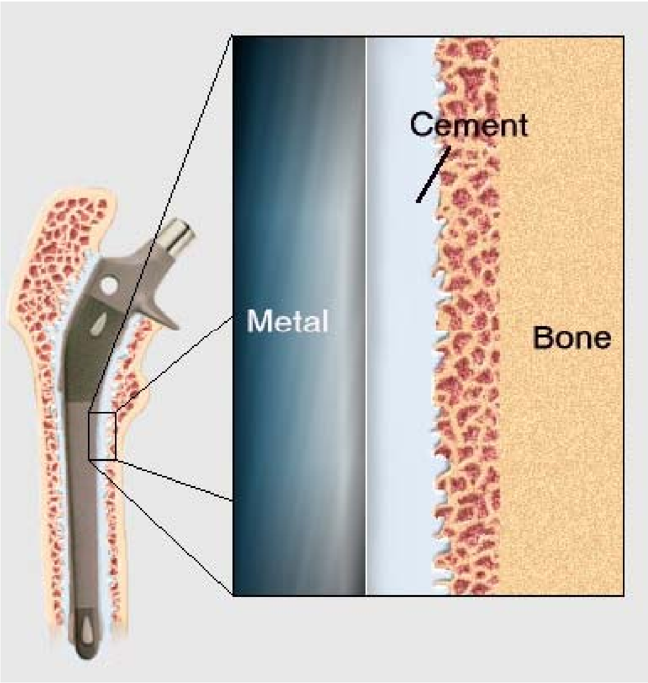
Final assembling



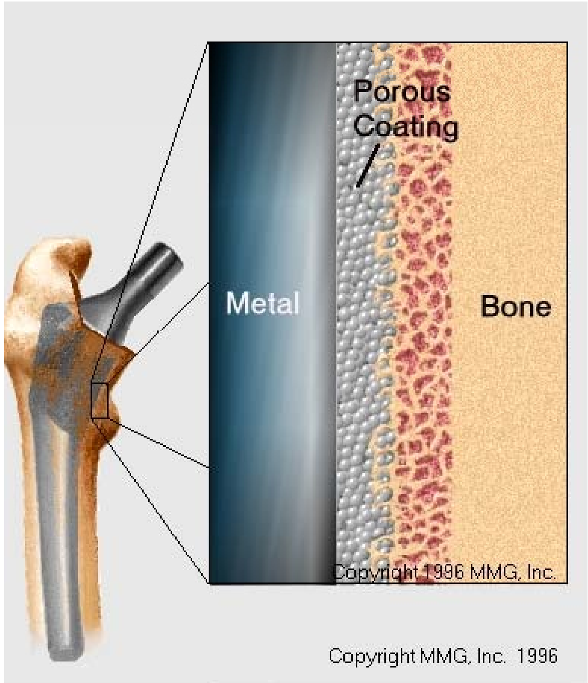


total hip arthroplasty – stem fixation

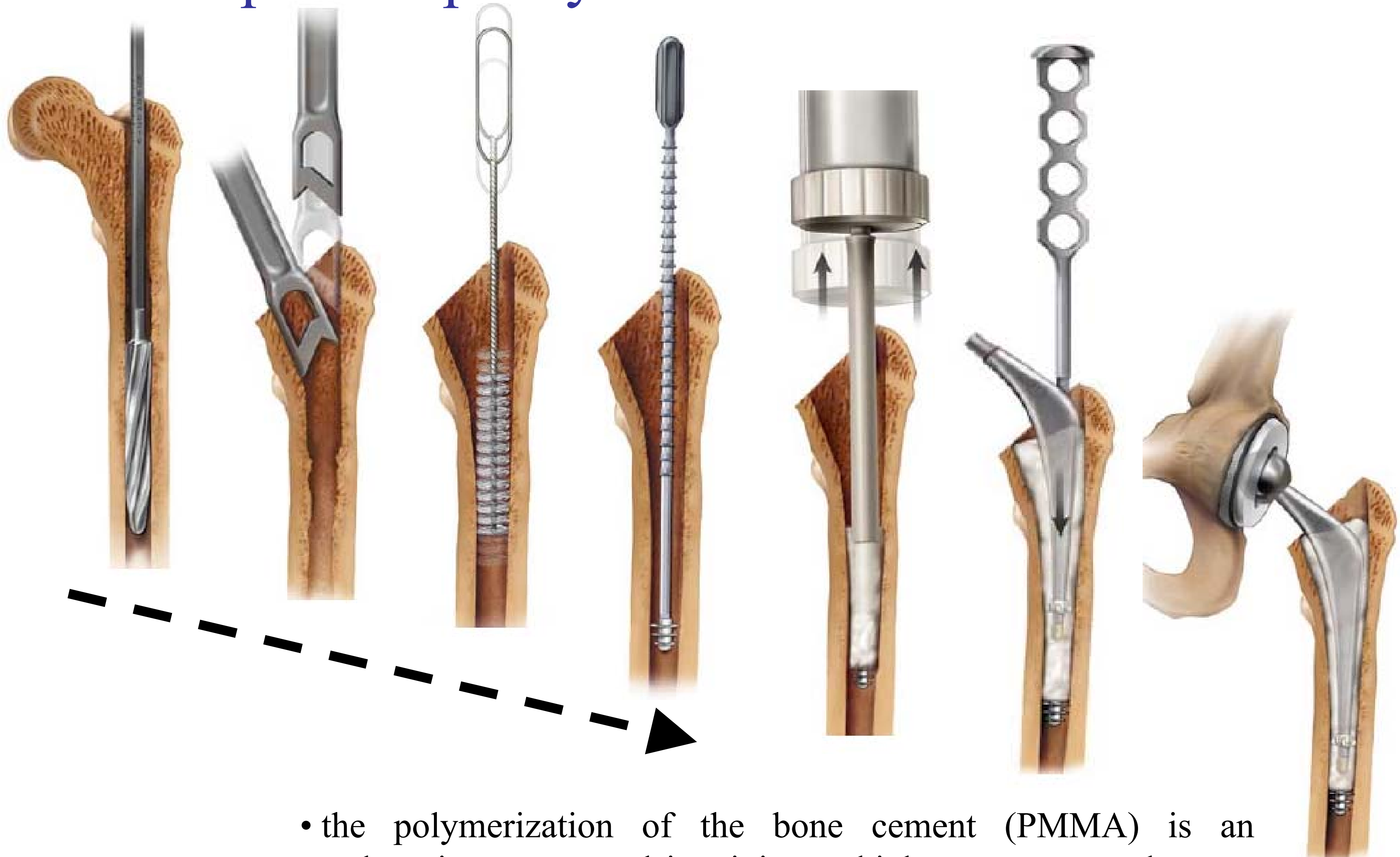
with bone cement



biological fixation

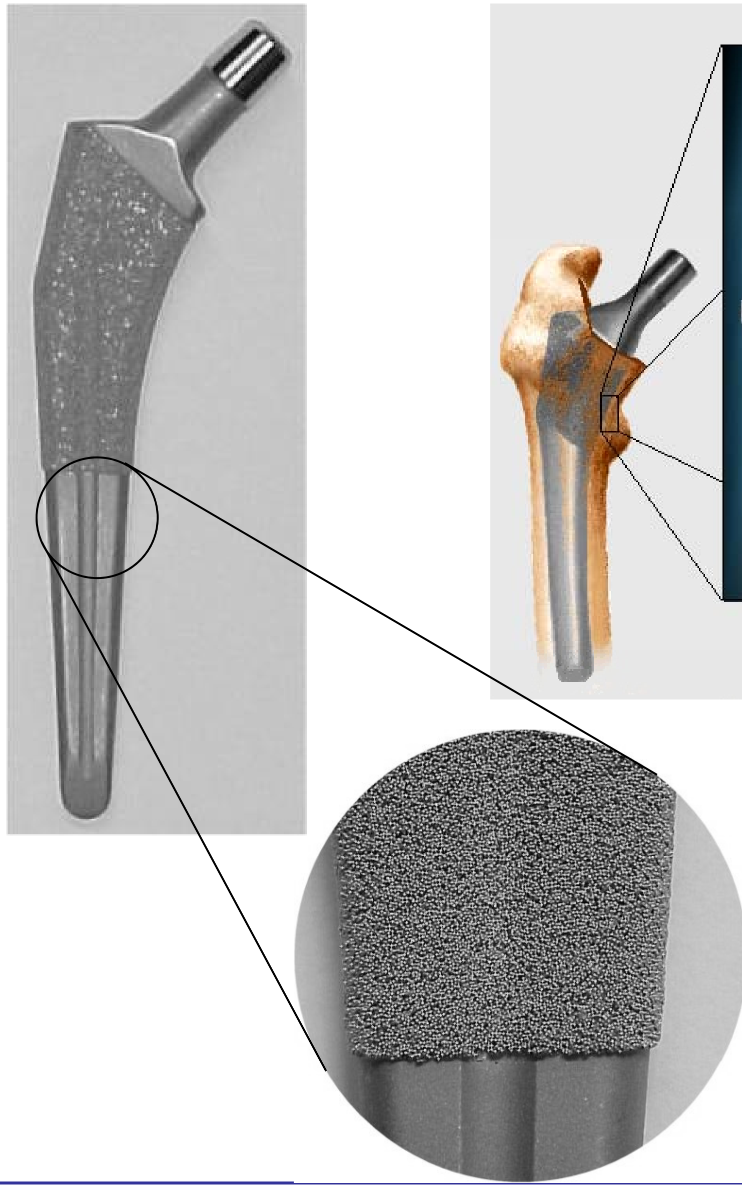


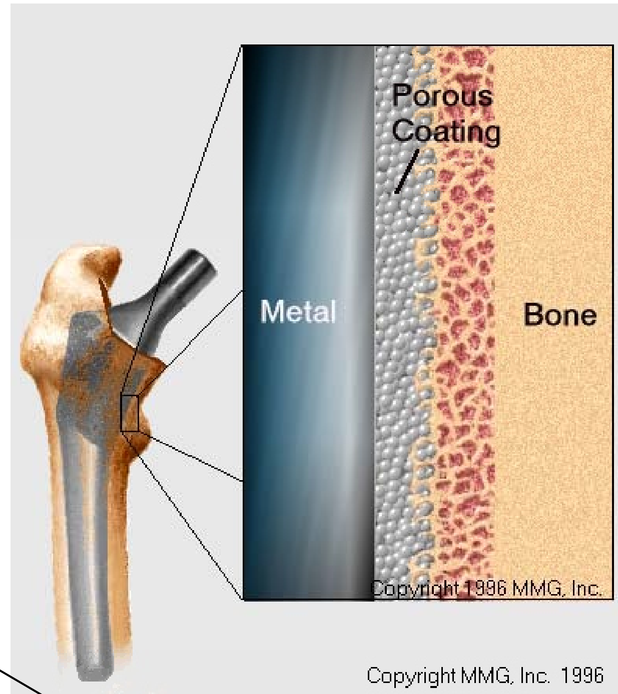
total hip arthroplasty – stem fixation with cement

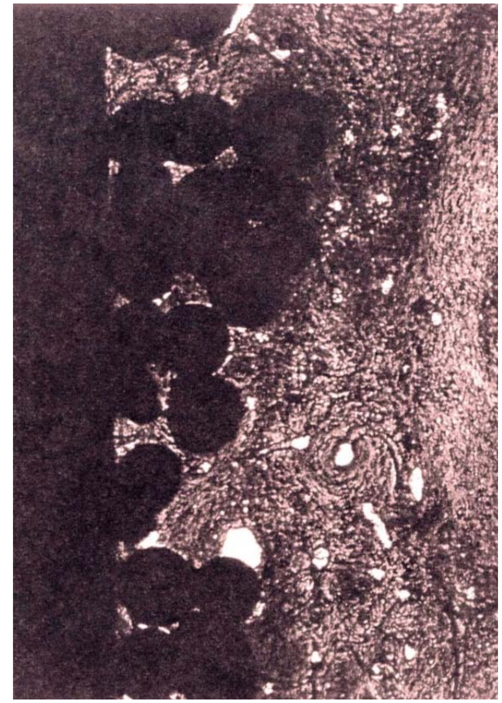


• the polymerization of the bone cement (PMMA) is an exothermic process and it originates high temperatures that can lead to bone necrosis.

total hip arthroplasty – biological stem fixation



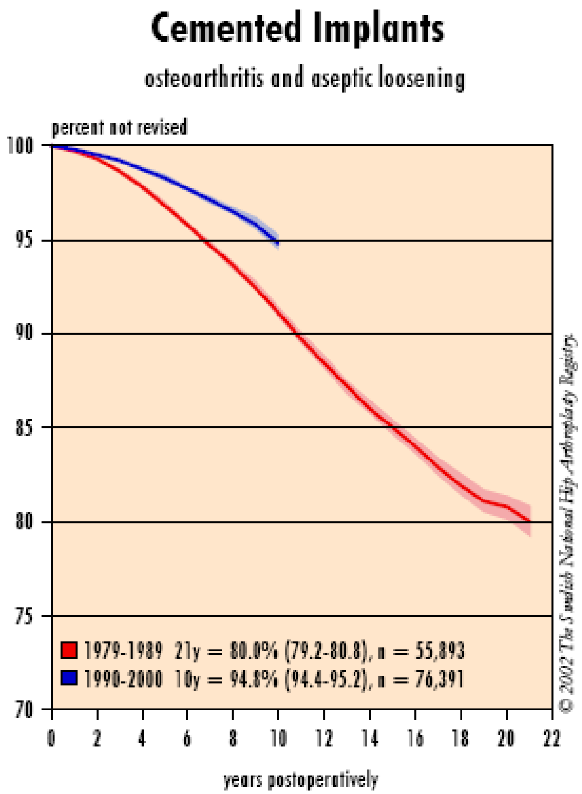


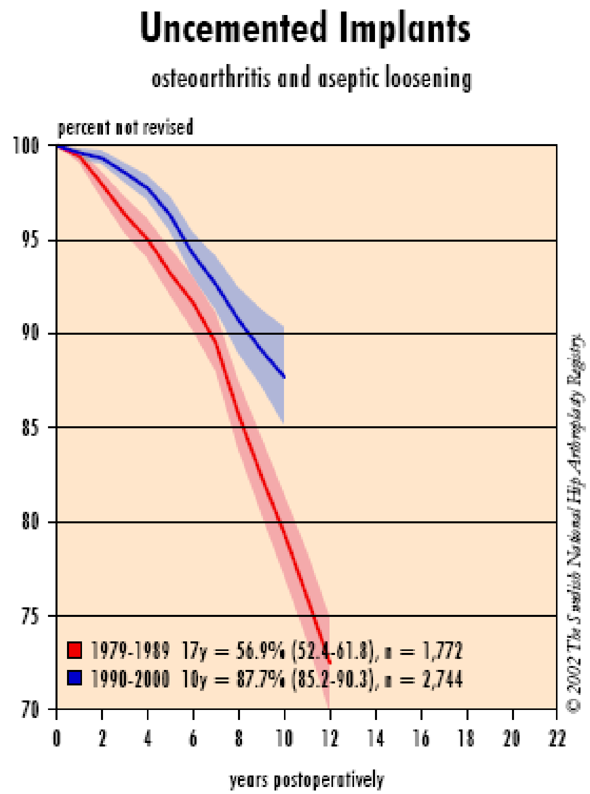


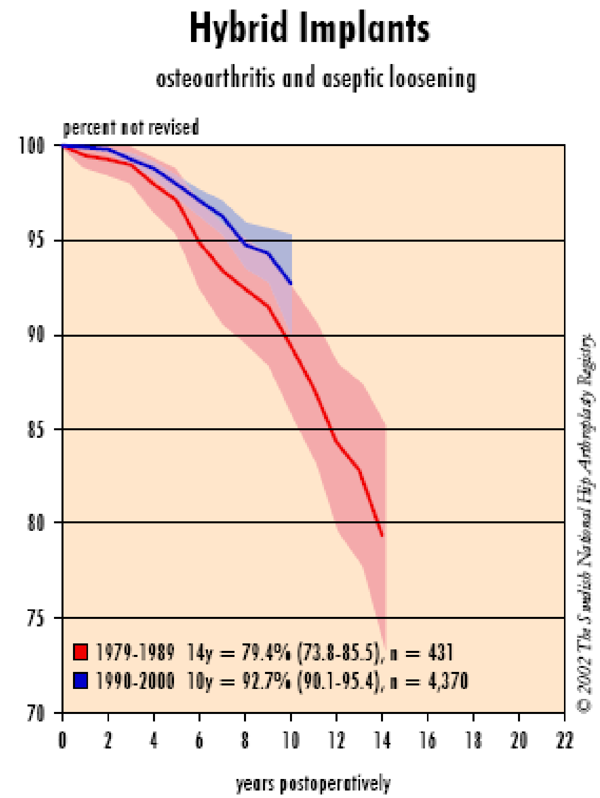
• immediately after the surgery there is no osteointegration.

• to obtain osteointegration (bone ingrowth), it is necessary to achieve suitable mechanical conditions. The stability of the stem (*i.e.*, small interface displacements) is one of the requirements.

total hip arthroplasty – revision rates

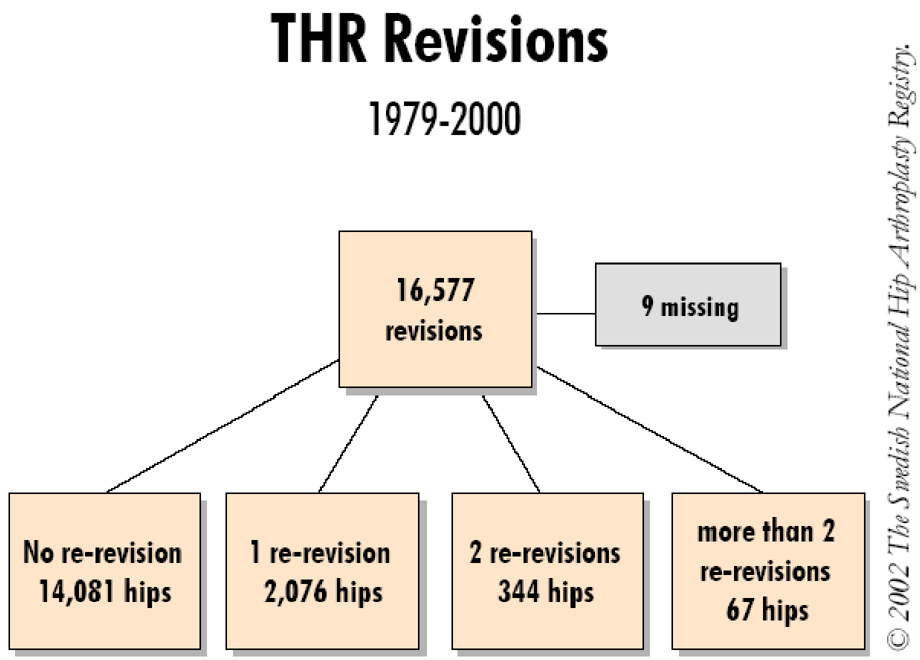






- for a moderate active patient, an hip implant can have a lifetime of 15 - 20 years → and for a more active patient (younger)?
- most of the patients with hip implant and moderate activity do not have pain in the first 10-15 years

Orthopaedic implants– revision



Reason for Revision

14,081 1st revision THR 1979-2000

Reason	N	Share
Aseptic loosening	10,610	75.4%
Primary deep infection	948	6.7%
Dislocation	810	1.5%
Fracture only	716	5.1%
Technical error	425	5.8%
Implant fracture	215	3.0%
Secondary infection	128	0.3%
Polyethylene wear	126	0.4%
Pain	46	0.9%
Miscellaneous	56	0.9%
Missing	1	0.0%
Total	14,081	100%

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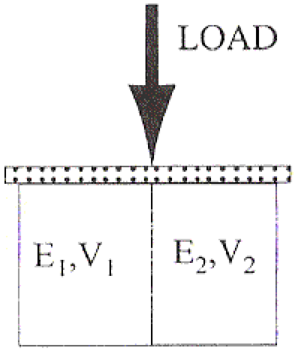
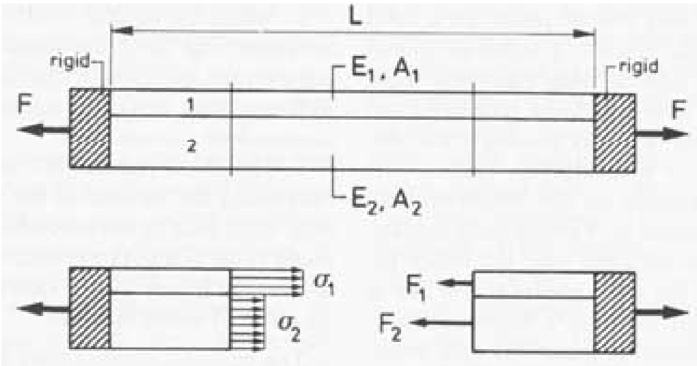
- one of the biggest problem is the aseptic loosening.
- another problem is the bone resorption around the implant that can lead to failure and make difficult the revision surgery.

Orthopaedic implants – load transfer

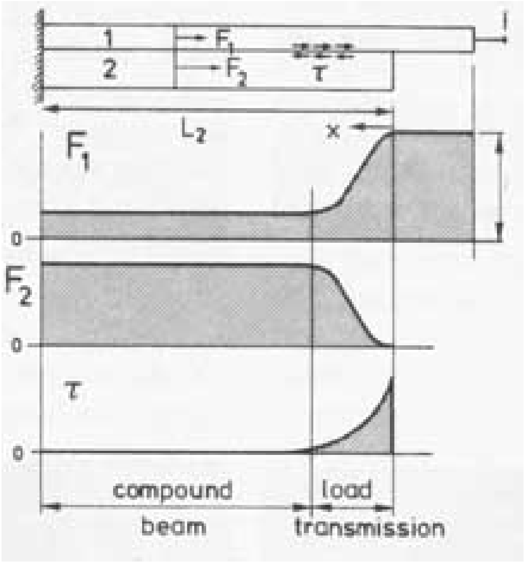


Voigt model → $E_{eq} = (A_1/A) \,.\, E_1 + (A_2/A) \,.\, E_2$

$F_1 = [(A_1.E_1)/(A_1.E_1 + A_2.E_2)]$, $F_2 = [(A_2.E_2)/(A_1.E_1 + A_2.E_2)]$



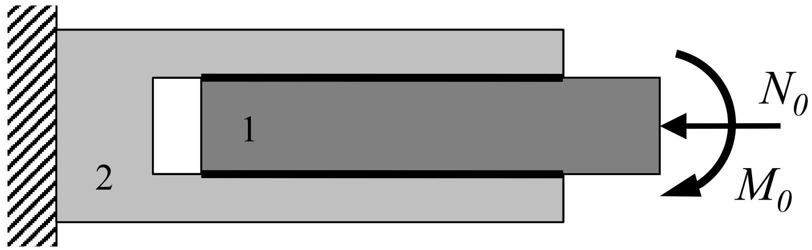
• the stiffest material supports more load.

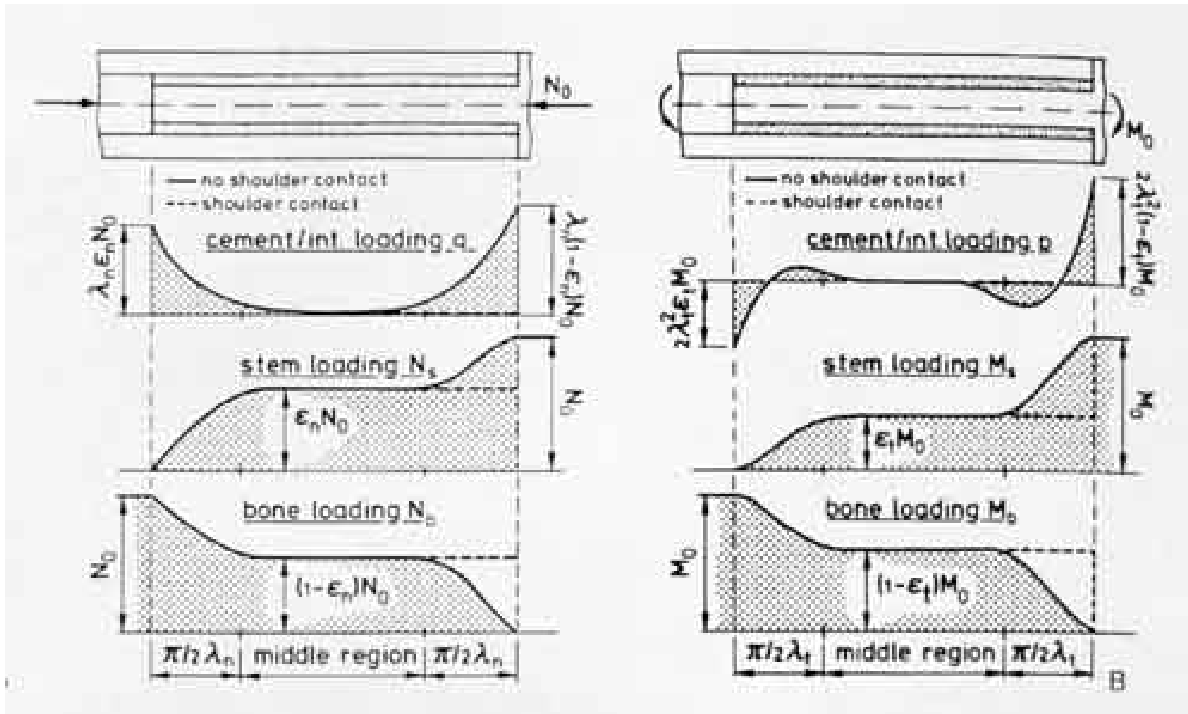


• before reach the situation described by voigt model, it is necessary to transfer the load from a component to the other one..

• the loading transfer is due to shear stress on the interface.

Orthopaedic implants – load transfer

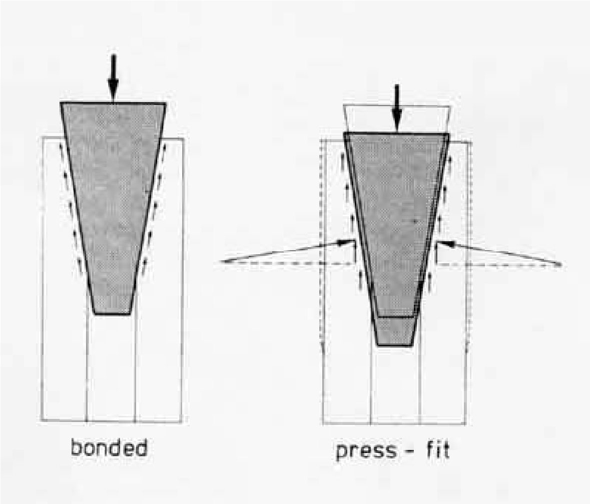




• there are some analogies between load transfer in bending and under axial loading.

• the load transfer between the components takes place near the ends of the interface.

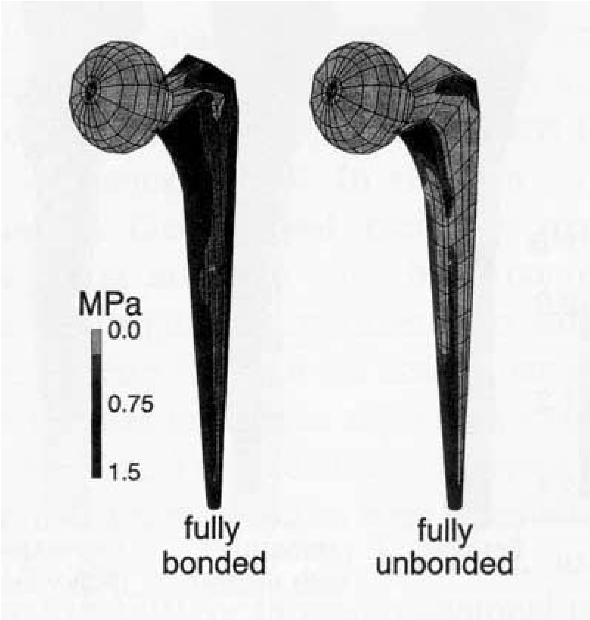
Orthopaedic implants – load transfer



• shear stress only exist if the interface is bonded or if there is friction between the two components

• if there is no shear stress, the stem subsidence will originate forces to support the stem.

• the fully bonded model is a model for ideal conditions.



• the obtained interface stress strongly depends on the interface conditions.

Orthopaedic implants – influence of the stem material

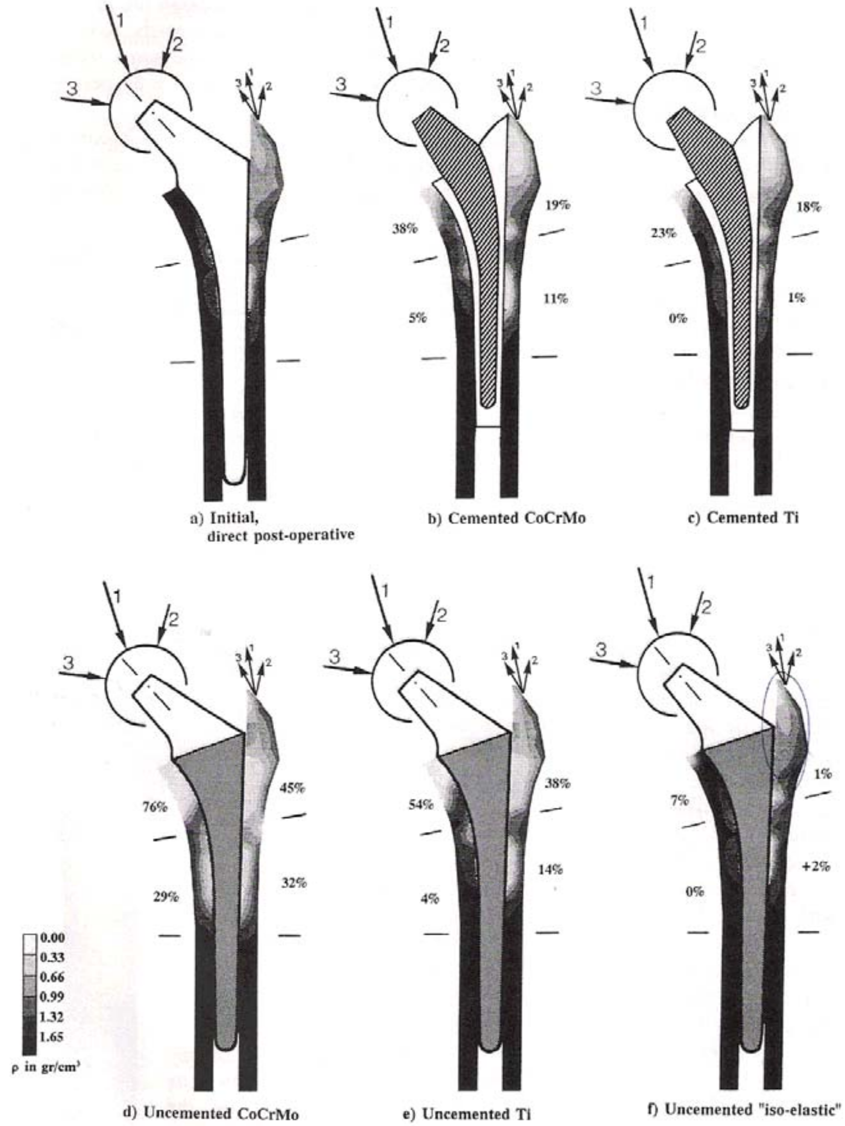


Fig. 6 *Density distributions of the proximal femur with different stem types. The percentages bone loss in four different area's (Gruen's zones) are indicated.*

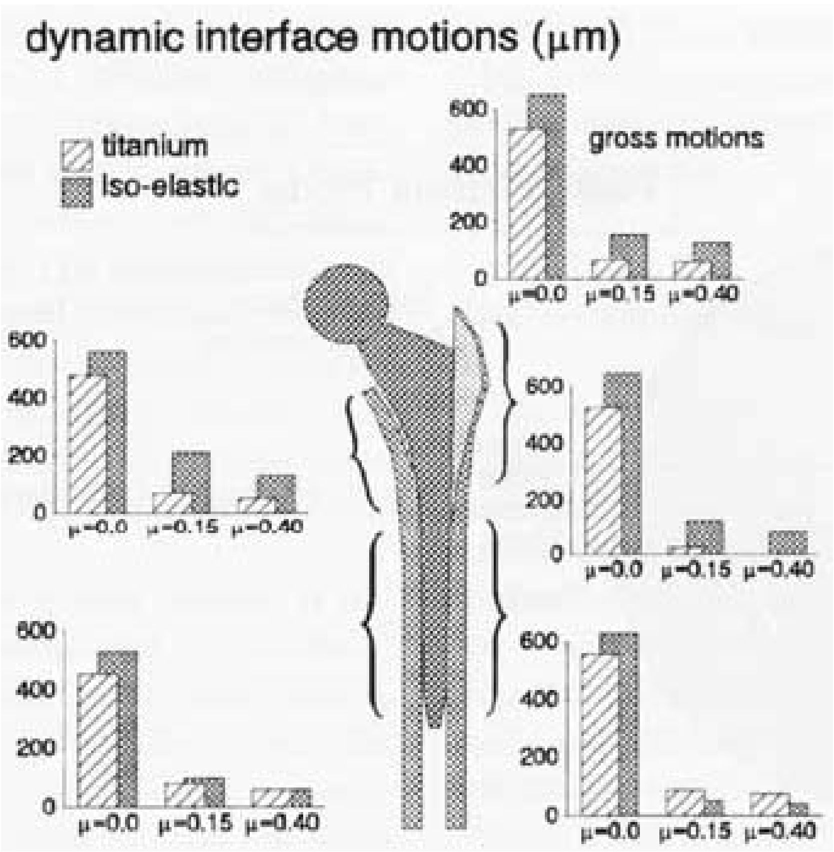
•The stem material is stiffer than bone. It leads to the *stress shielding* effect and consequently to the bone resortpion.

• more stiff stems originate more *stress shielding*, and thus more bone resorption.

• the bone cement is less stiff than the bone and the stem.

• it is possible to analyze the cemented stem assuming the stem/cement set as a single component with an intermediate (equivalent) stiffness.

Orthopaedic implants – influence of the stem material



• the interface displacement on the interface is an important issue for the implant analysis.

• more flexible (compliant) stems are subjected to higher interface displacements.

• higher displacement on the interface lead to a high loosening rate.

• a correct choice of the stem stiffness should take in account these different aspects, and a compromise solutions must be achieved.

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