Bone Tissue Mechanics

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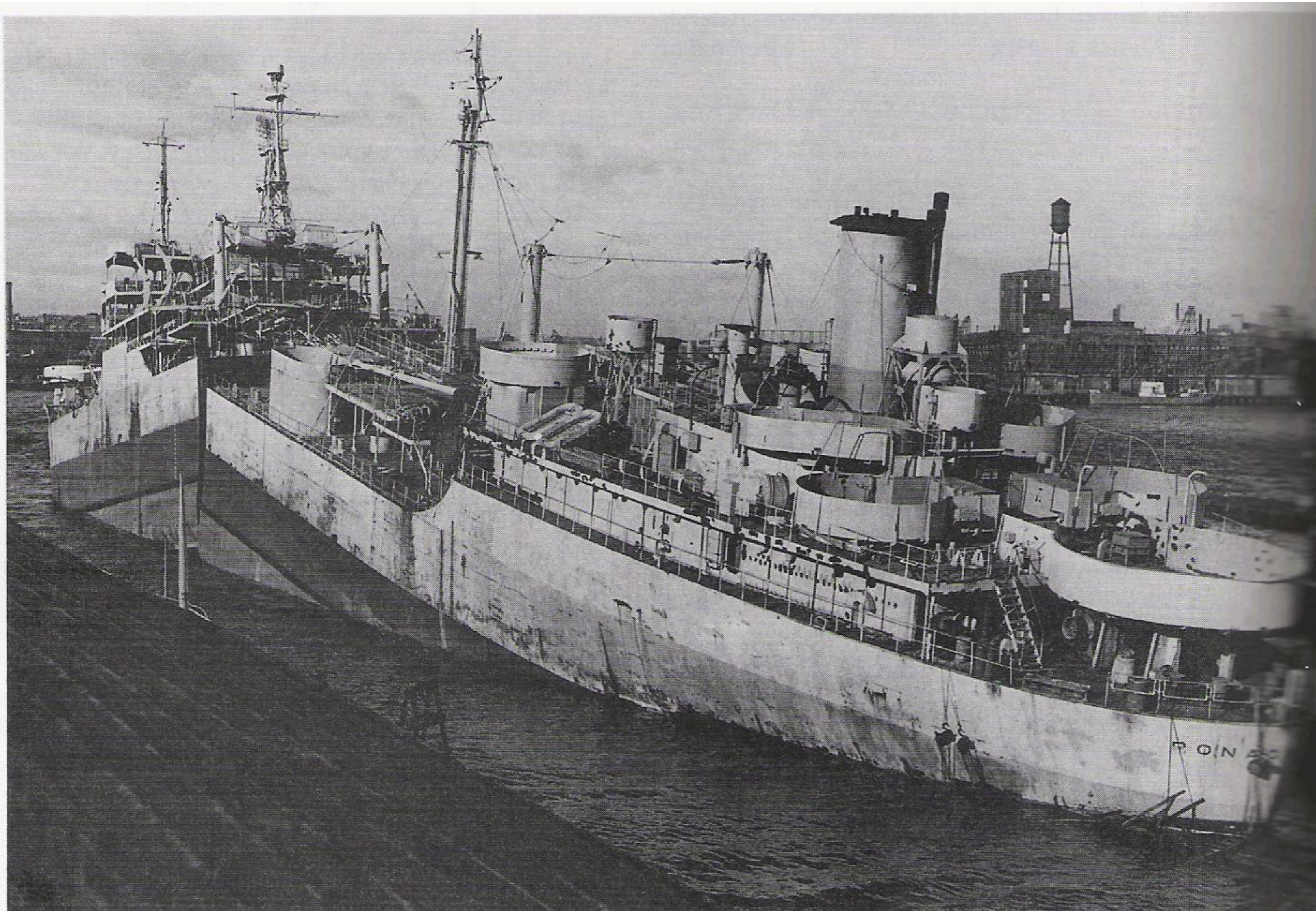
PART 6

Introduction to linear elastic fracture mechanics

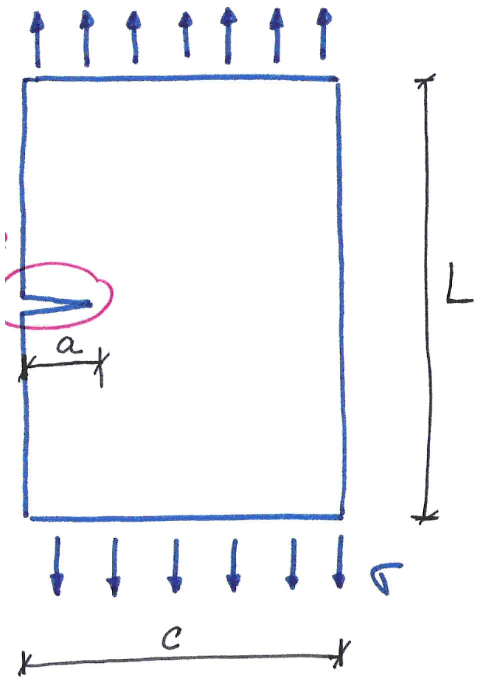


The hull of this T-2 tanker cracked in two while the ship was moored in calm water.

Introduction to linear elastic fracture mechanics

σ (nominal stress)

Crack
dimension
a



• The state of stress at the crack tip is complex

• The analysis is based on the stress intensity factor K

$$\boxed{K = \beta \sigma \sqrt{\pi a}}$$

β is a parameter that depends on geometry and loading

The unit for K is $MPa\sqrt{m}$

• Fracture occurs if the stress intensity factor K is higher then the material K_c (fracture toughness)

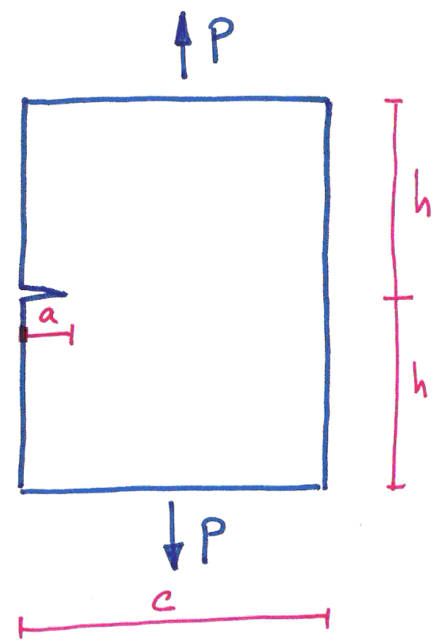
Example

$a = 15$ mm

$c = 100$ mm

$t = 20$ mm

thickness



data

$$K = \beta \sigma \sqrt{\pi a}$$

$$\beta = \left[1{,}12 - 0{,}23\left(\tfrac{a}{c}\right) + 10{,}6\left(\tfrac{a}{c}\right)^2 - 21{,}7\left(\tfrac{a}{c}\right)^3 + 30{,}4\left(\tfrac{a}{c}\right)^4\right]$$

(erro < 1% se $a/c \leq 0{,}6$, $h/c > 1$)

		σ_U (MPa)	σ_Y (MPa)	K_C (MPa$\sqrt{m}$)
Steel	C	1290	1150	77
Steel	D	1600	1410	50

a) For the material type C, determine the value of failure load P ?

b) If the same load is applied to the material D, what is the critical crack length?

Solution:

a)

critical situation $\longrightarrow \boxed{K_I = K_c} \Rightarrow \beta \sigma \sqrt{\pi a} = K_c$

$$\Rightarrow \left[1{,}12 - 0{,}23\left(a/c\right) + 10{,}6\left(a/c\right)^2 - \ldots\right]\sigma\sqrt{\pi a} = K_c$$

$a = 15$ mm
$c = 100$ mm
$a/c = 0{,}15$

$K_c = 77$ MPa$\sqrt{m}$

$$\longrightarrow \left[1{,}12 - 0{,}23 \times 0.15 + 10{,}6\,(0{,}15)^2 - \ldots\right]\sigma\sqrt{\pi \times 15\times10^{-3}} = 77$$

$$\Rightarrow \sigma = \frac{77}{\left[1{,}12 - 0{,}23\times 0{,}15 + 10{,}6\times(0{,}15)^2 - \ldots\right]\sqrt{\pi\times15\times10^{-3}}}$$

$$\Rightarrow \boxed{\sigma = 280 \text{ MPa}}$$ ⟵ nominal stress

nominal stress:

$$\sigma = \frac{F}{A} \Rightarrow F = \sigma A = 280\times10^{6}\times\left(100\times10^{-3}\times 20\times10^{-3}\right) = 280\times100\times20$$

$$\Rightarrow \boxed{F = 560 \text{ KN}}$$

b) Material D ($K_c = 50$ MPa$\sqrt{m}$), $\sigma = 280$ MPa

<u>critical situation</u> $\longrightarrow$ $K = K_c \Rightarrow \beta \sigma \sqrt{\pi a} = K_c$

$$\Rightarrow \left[1{,}12 - 0{,}23\left(a/c\right) + 10{,}6\left(a/c\right)^2 - 21{,}7\left(a/c\right)^3 + 30{,}4\left(a/c\right)^4\right]\sigma\sqrt{\pi a} = K_c$$

unknown $\longrightarrow a$?

Data: $c = 100$ mm, $\sigma = 280$ MPa, $K_c = 50$ MPa$\sqrt{m}$

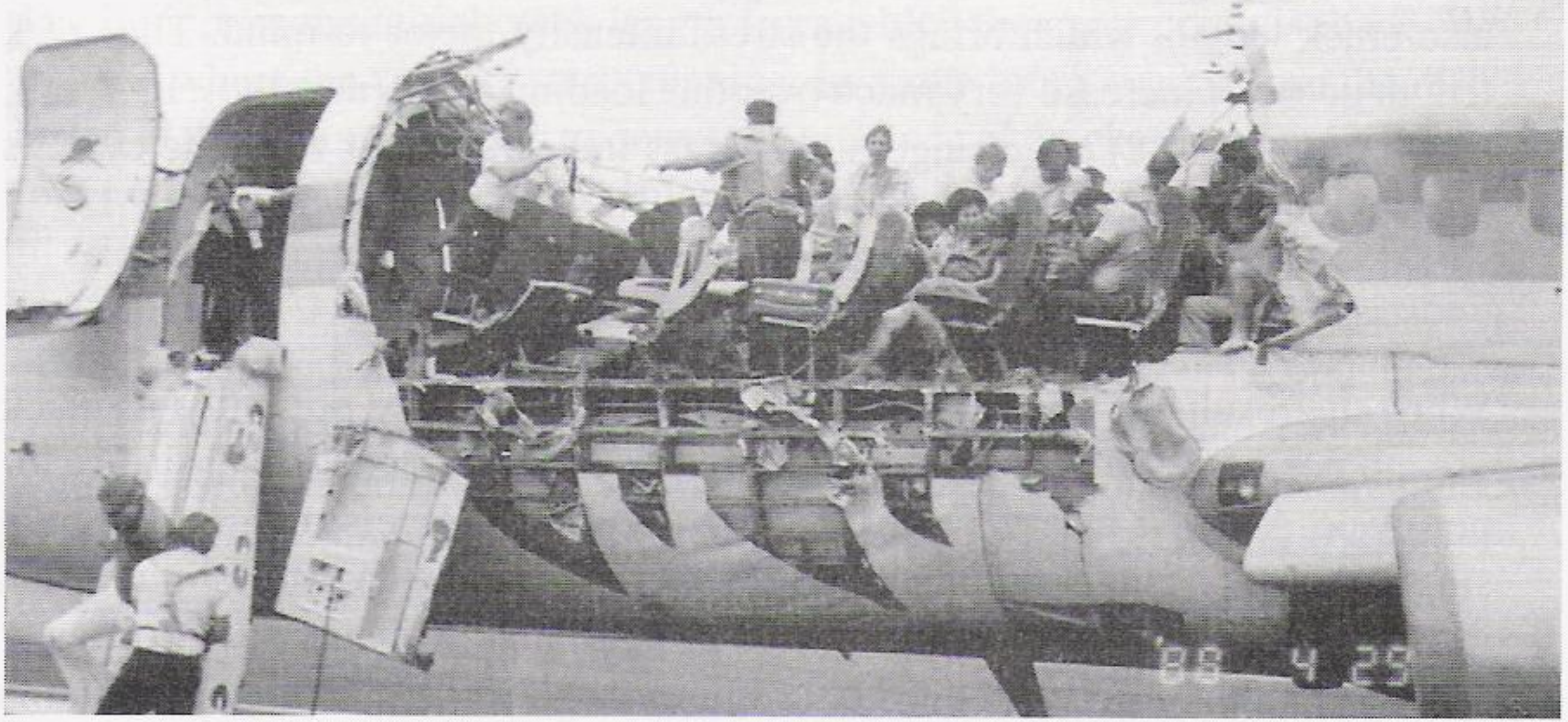
Solving the equation:

$$\left[1{,}12 - 0{,}23\left(\frac{a}{0{,}1}\right) + 10{,}6\left(\frac{a}{0{,}1}\right)^2 - 21{,}7\left(\frac{a}{0{,}1}\right)^3 + 30{,}4\left(\frac{a}{0{,}1}\right)^4\right] \times 280 \times \sqrt{\pi a} = 50$$

$$\xrightarrow[\text{solucoO}]{} a = 7{,}6 \times 10^{-3} \text{ m}$$

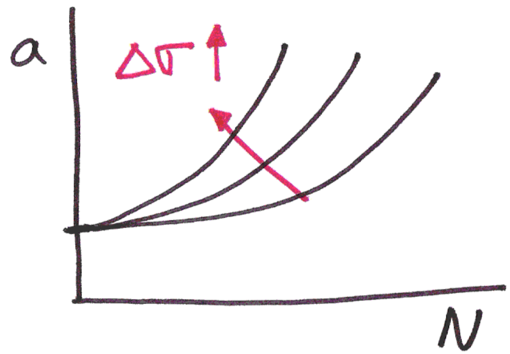
$$\boxed{a = 7{,}6 \text{ mm}}$$

Crack propagation analysis



An Aloha Airlines Boeing 737 lands safely despite the loss of its fuselage.

Crack propagation



For cyclic loads cracks propagate for values of $K < K_c$

$$\Delta\sigma = \sigma_{max} - \sigma_{min}$$

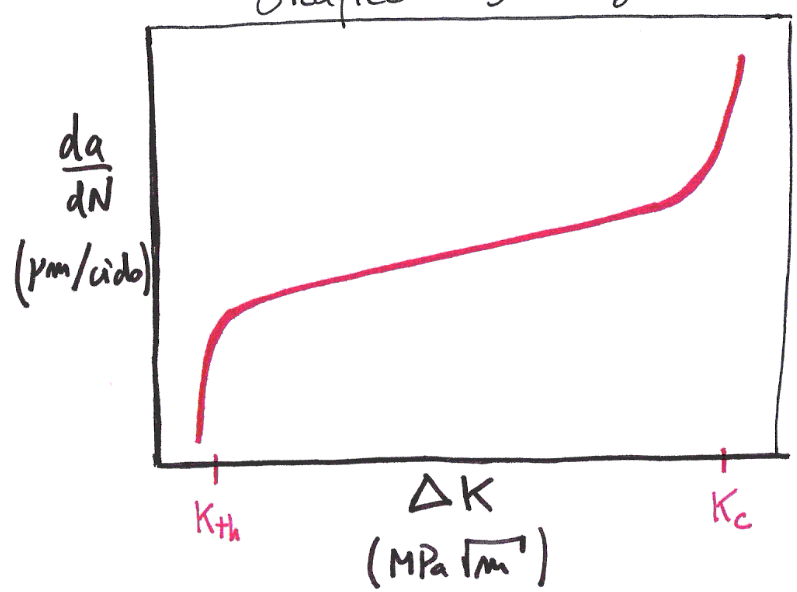
$$\Delta K = K_{max} - K_{min}$$
$$= (\sigma_{max} - \sigma_{min})\,\beta\sqrt{\pi a} = \Delta\sigma\,\beta\sqrt{\pi a}$$

Lei de Paris

$$\frac{da}{dN} = A(\Delta K)^n \Rightarrow dN = \frac{da}{A(\Delta K)^n}$$

$$\Rightarrow \int_0^N dN = \boxed{N = \frac{1}{A}\int_{a_0}^{a} \frac{da}{(\Delta K)^n}}$$

Gráfico log vs log



Problem

-A cylinder with a diameter of 12,7 mm is subject to an alternating tension load between 0 and 44,5 kN.

- the cylinder has a crack with initial length of a_0 = 0,254 mm
- for this geometry and loading $K = 1{,}12\,\sigma\sqrt{\pi a}$
- the material property is $K_c = 25{,}3\ MPa\sqrt{m}$
- the parameter to apply the Paris Law are: A= 19,1x10^{-12} and n=3.

How many cycles are needed to reach failure?

Solution

The alternating stress is:

$$\sigma_{max} = \frac{F_{max}}{A} = \frac{F_{max}}{\pi\left(\frac{d}{2}\right)^2} = \frac{44{,}5\times10^3}{\pi\left(\frac{12{,}7\times10^{-3}}{2}\right)^2} = 351\times10^6\ \frac{N}{m^2} = 351\ MPa$$

$$\sigma_{min} = \frac{F_{min}}{A} = 0$$

ΔK is:

$$\Delta K = K_{max} - K_{min} = 1{,}12\,(\sigma_{max} - \sigma_{min})\sqrt{\pi a}$$

$$= 1{,}12\,(351 - 0)\sqrt{\pi a} = 393{,}1\sqrt{\pi a}$$

The critical length of the crack a_c is:

$$K = K_c \Rightarrow 1{,}12\,\sigma_{max}\sqrt{\pi a} = K_c \Rightarrow 1{,}12\times351\sqrt{\pi a} = 25{,}3$$

$$\Rightarrow a_c = \left(\frac{25{,}3}{1{,}12\times351}\right)^2\times\frac{1}{\pi} = 1{,}32\times10^{-3}\,m \Rightarrow \boxed{a_c = 1{,}32\ mm}$$

The number of cycles to reach failure is:

$a_0 = 0{,}254 \text{ mm} = 0{,}254 \times 10^{-3} \text{ m} \longrightarrow N_0 = 0$

$a_c = 1{,}32 \text{ mm} = 1{,}32 \times 10^{-3} \text{ m} \longrightarrow N_c$

<u>Paris Law</u> $\frac{da}{dN} = A(\Delta K)^n \Rightarrow dN = \frac{1}{A}\frac{da}{(\Delta K)^n} \Rightarrow \int_{N_0=0}^{N_c} dN = \frac{1}{A}\int_{a_0}^{a_c}\frac{da}{(\Delta K)^n}$

$$N_c = \frac{1}{A}\int_{a_0}^{a_c}\frac{da}{(\Delta K)^n} = \frac{1}{19{,}1\times 10^{-12}}\int_{0{,}254\times 10^{-3}}^{1{,}32\times 10^{-3}}\frac{da}{(393{,}1\sqrt{\pi a})^3} \simeq 10\times 10^3 \text{ ciclos}$$

$N \simeq 10$ thousand cycles

Fatigue and Fracture Resistance of Bone

Bone Failure – Fatigue and Creep

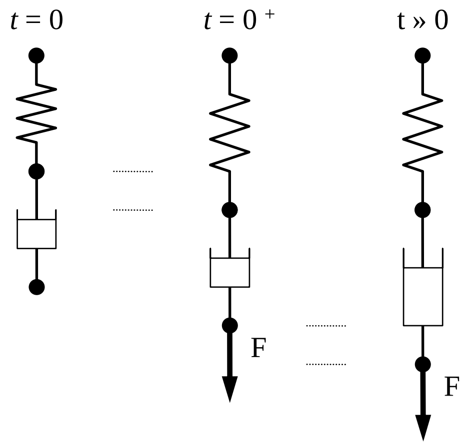
• bone does not fail only under loads large enough to exceed the failure stress (monotonic fracture).

• different loads, <u>corresponding to stress below the failure stress</u>, can lead to damage accumulation, and gradually lead to bone failure.

• There are two ways for this failure, fatigue and creep (sometimes working together)

  • fatigue is the result of cyclic loads (that not exceed the failure stress)

  • in creep the load is applied continuously (not exceeding the failure stress).



damage – micro cracks

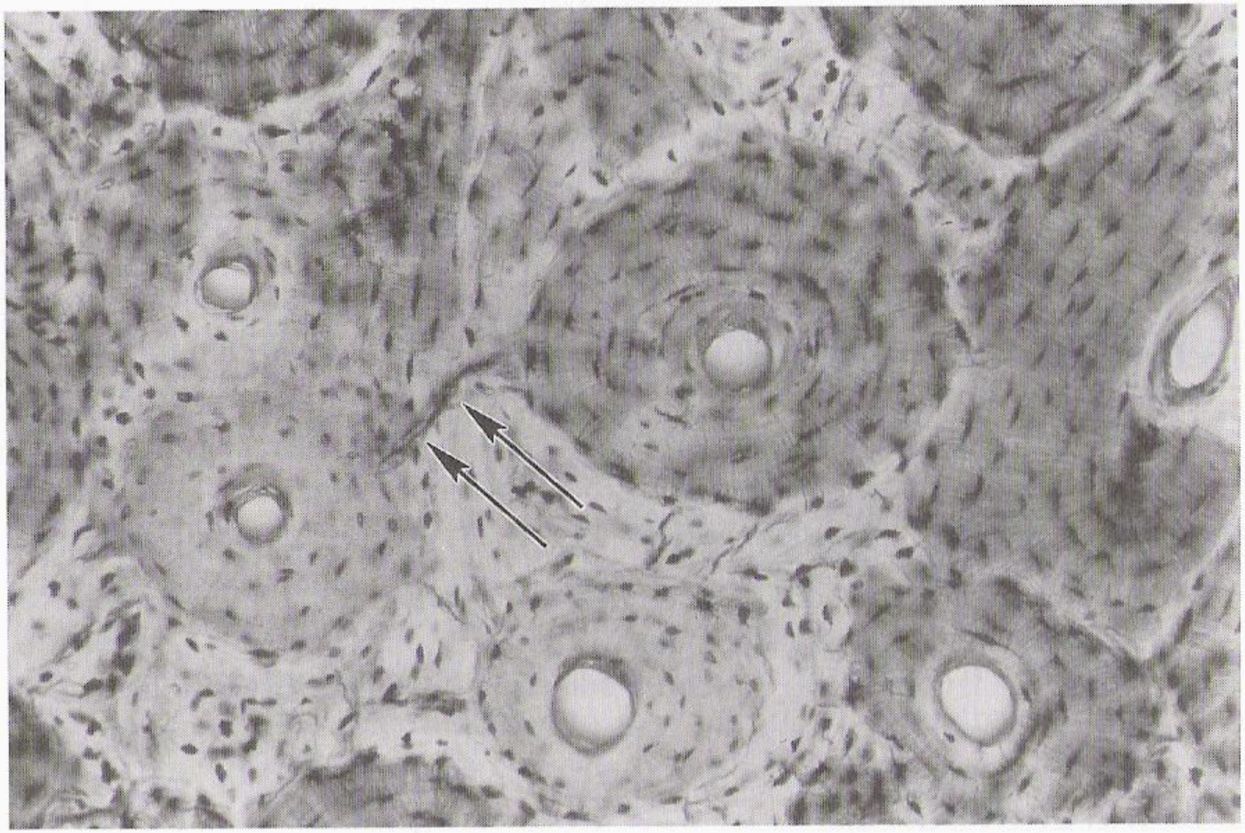
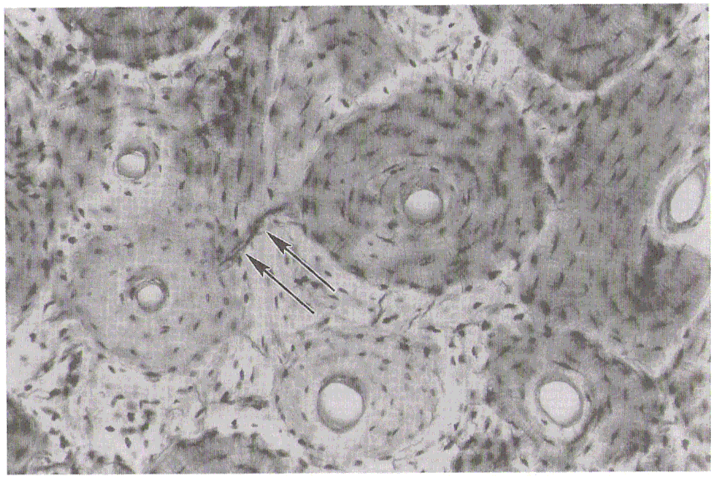


FIGURE 5.1. A typical microcrack *(arrows)* as seen in a 100-μm-thick histologic cross section of human cortical bone. The specimen was stained in basic fuchsin en bloc to mark the surfaces of cracks (such as this one, which would be red in a color photograph) existing before the section was cut. Artifactual cracks produced by the sectioning process are not stained (Burr and Stafford, 1990). this crack is about 80 μm long in the plane of the section, and its ends abut the cement lines of two secondary osteons. Most microcracks in cortical bone are of this type.

damage – micro cracks



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• damage due to fatigue/creep can be observed in healthy bones, in the form of microcracks.

• these microcracks can, under certain conditions, growth in number, as well as growth in length and become macrocracks that can lead to bone failure.

• in general, these microcracks are removed by bone remodeling.

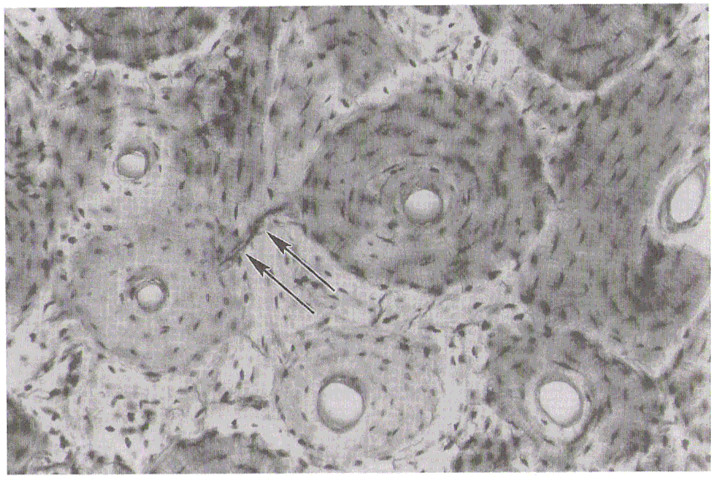
• when the damage accumulation is faster then bone remodeling a "stress fracture" can occur. (elite athletes, soldiers, ballet dancers, etc…)

• the crack propagation is also controlled by the lamellar structure of bone.

• in general, cracks appear due to an overloading or cyclic loading.

• can be also some congenital defect or introduced during a surgical procedure.

damage – micro cracks



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- the damage accumulates exponentially, and more rapidly in woman than in men, after the 40 years old.
- Burr and Stafford (1990) counted 0.014 microcracks /mm^2 in the ribs of a 60 years-old man.
- Wenzel et al. (1996) found approximately 5 microcracks/mm^2 in the trabecular bone from the spine.
- About 80%–90% of micro cracks in cortical bone are found in the interstitial matrix between

The effect of age in microdamage

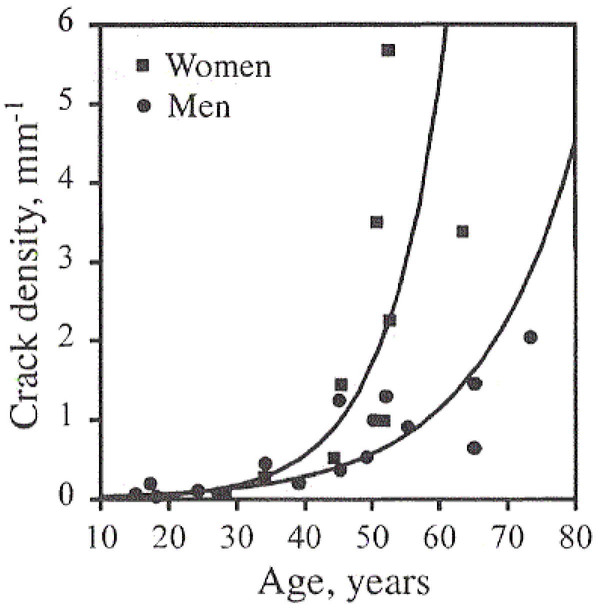


FIGURE 5.20. Microcrack density vs. age for men *(circles)* and women *(squares)*. (Reproduced from Schaffler et al., 1995.)

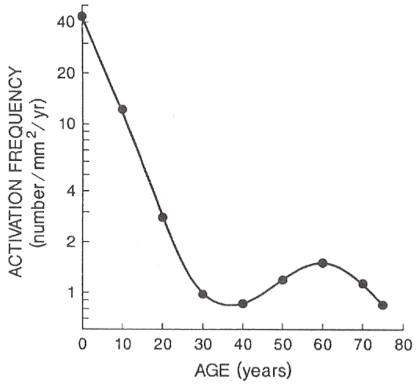


FIGURE 2.21. BMU activation rate vs. age for human ribs. (From data by Frost, 1964b.)

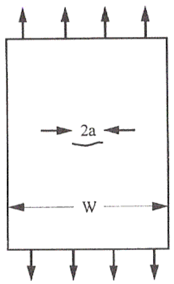
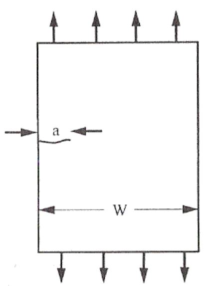
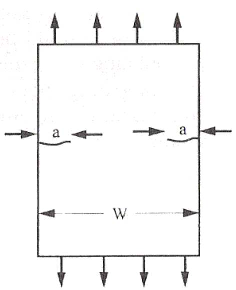
- the evolution of microcracks with age is exponential .
- after the 40 year-old the crack density is higher in woman.
- probably this is related with the reduction of bone mass. increasing bone strain, and probably fatigue damage is sensitive to the strain magnitude.
- many nontraumatic fractures in the elderly (osteoporotic fractures) are essentially stress fractures.

- remember the BMU activation with age, namely after 40 y-old (is there any relation?)

Fracture mechanics – introduction

Linear elastic fracture mechanics, G.R. Irwin (1957)

TABLE 5.2. Equations for the constant C in three simple cases

Crack geometry	C
	Plate of width W in tension, Mode I crack of length $2a$ in center. $C = [1 - 0.1(a/W)^2 + 0.96(a/W)^4]\,[\sec(\pi a/W)]^{1/2}$
	Plate of width W in tension, Mode I crack of length a on one edge. $C = [(sW/\pi a)\tan(\pi a/2W)]^{1/2} \bullet \dfrac{0.752 + 2.02(a/W) + 0.37[1 - \sin(\pi a/2W)]}{\cos(\pi a/2W)}$
	Plate of width W in tension, Mode I crack of length a on each edge. $C = [1 + 0.122\cos^4(\pi a/W)]\,[(W/\pi a)\tan(\pi a/W)]^{1/2}$

- to measure the severity of a crack the ***stress intensity factor*** K is used,

$$K = C.s.(\pi.a)^{½} \qquad [K]\text{=MPa.m}^{½}$$

where s is the nominal stress, a is the crack length and C is a coefficient related with the crack geometry and mode of loading

- 3 different kinds (modes) of cracks:

Mode I – tension

Mode II - shear

Mode III – tearing

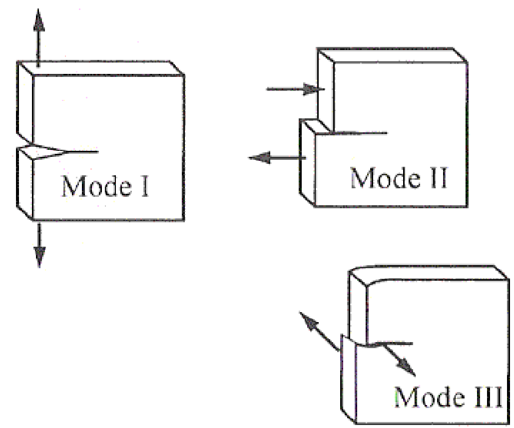


FIGURE 5.3. Modes of cracking.

Fracture mechanics – introduction

• if for a given load the stress intensity factor, K, is greater than a critical value, K_c (the values supported by the material) then the crack will propagate.

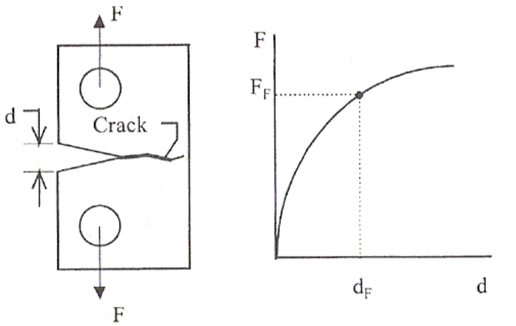
• the maximum value support by the material (critical value), K_c, is the fracture toughness.

TABLE 5.1. Fracture toughness for several materials

Material	K_{Ic}, MPa-m$^{1/2}$
2024 Aluminum	20–40
4330V Steel	86–110
Ti-6Al-4V	106–123
Concrete	0.23–1.43
Al_2O_3 ceramic	3–5.3
SiC ceramic	3.4
PMMA polymer	0.8–1.75
Polycarbonate polymer	2.75–3.3
Cortical bone	2.2–6.3

• note the low fracture thougness of bone cement (PMMA)

Fracture mechanics – introduction



n-
sion to drive a crack to the *right* as shown. The tensile force F is plotted against the crack opening displacement, d. A failure point, $P(F_f, d_f)$, on the curve is defined by certain rules, and K_{Ic} is calculated from F_f and d_f using a formula.

- cracks in long bones propagate in a direction parallel to the bone axis.
- when a crack propagates transversally (perpendicular to lamellar structure) the crack tends to change the direction and propagate along the principal axis of the bone.
- the fracture toughness of bone is anisotropic .

TABLE 5.3. Experimental values of K_c and G_c for bone

Bone Type	K_c, MPa-m$^{1/2}$	G_c, Jm^{-2}	Source
Mode I, transverse fracture			
Bovine femur	5.49	3100–5500	Melvin and Evans, 1973
Bovine tibia	2.2–4.6	780–1120	Bonfield and Datta, 1976
Equine metacarpus	7.5	2340–2680	Alto and Pope, 1979
Human tibia	2.2–5.7	350–900	Norman et al., 1992
Mode I, longitudinal fracture			
Bovine femur	3.21	1388–2557	Melvin and Evans, 1973
Bovine tibia	2.8–6.3	630–2238	Behiri and Bonfield, 1984
Human femur	2.2–5.7	350–900	Norman et al., 1992
Mode II			
Human tibia	2.2–2.7	365	Norman et al., 1992

Fracture mechanics – introduction

Example:

• for s human tibia, K_{Ic}=4 MPa.m^{½}

• for a circular crack with radius a on the tension surface of a long bone under bending, the stress intensity factor K_I can be approximated by,

$$K_I = 2\times 1.025.s.(a/\pi)^{½}$$

• assuming for the elastic modulus E=20 GPa and for strain ε=2000 με (it is a physiologic strain level), the applied stress is:

$$s = E.\varepsilon = 20\times 10^{9} \times 2000\times 10^{-6} = 40 \text{ MPa}$$

• $K_I = K_{Ic} \Rightarrow 2\times 1.025.s.(a_c/\pi)^{½} = K_{Ic} \quad \Rightarrow 2\times 1.025 \times 40 \times (a_c/\pi)^{½} = 4$

$$\Rightarrow a_c = [4 / (2\times 1.025 \times 40)]^2 \times \pi \qquad \Rightarrow a_c = 7.5 \text{ mm}$$

• so the critical length for the crack is, a_c = 7.5 mm, that is a very high value.

Fatigue behavior of bone –S-N curve

• the bone behavior to cyclic loads is analog to other materials used in engineering – the fatigue process diminish the elastic modulus. The logarithm of the fatigue life (number of cycles until failure) is linearly proportional to the logarithm of stress (alternating stress) and cracks appears resulting from fatigue damage.

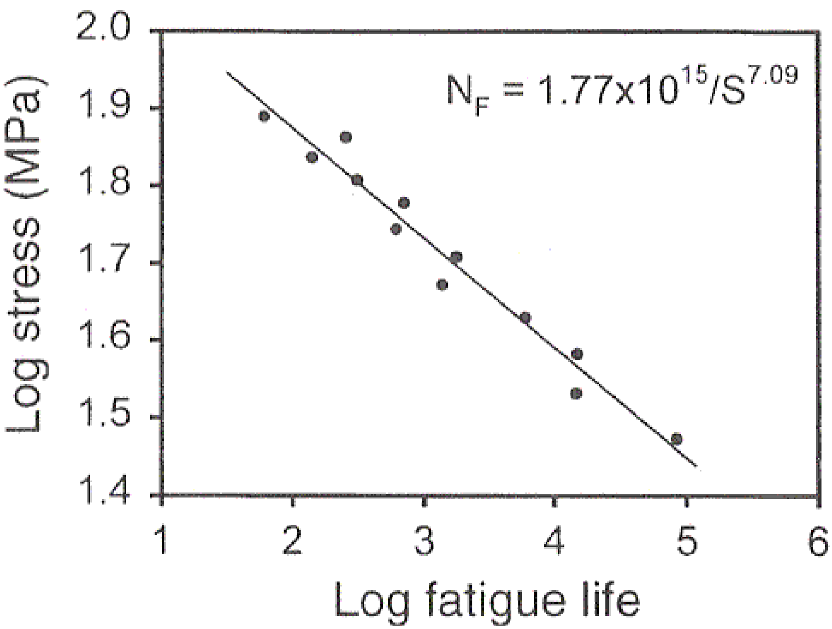


FIGURE 5.7. An early S-N curve for bone with its regression equation, for which $R^2 = .96$. (Replotted from data by King and Evans, 1967.)

• $N_F = c / S^q$, where c is a coefficient and q is a number between 5 and 15 depending on the type of bone and mode of loading.

•For the human femur (cortex) unden uniaxial fatigue load at 2 Hz,

tension → $N_F = 1.445\times10^{53} / S^{14.1}$

compression → $N_F = 9.333\times10^{40} / S^{10.3}$

where S is normalized by the initial elastic modulus. Thus S is also a strain and the units is $\mu\varepsilon$.

• These data was obtain for superphysiologic strain (2600–6600 $\mu\varepsilon$).

• Extrapolating for physiologic levels of strain (2000 $\mu\varepsilon$) the fatigue life would be 4.1 millions of cycles in tension and 9.3 millions in compression.

Fatigue behavior of bone – Elastic Modulus

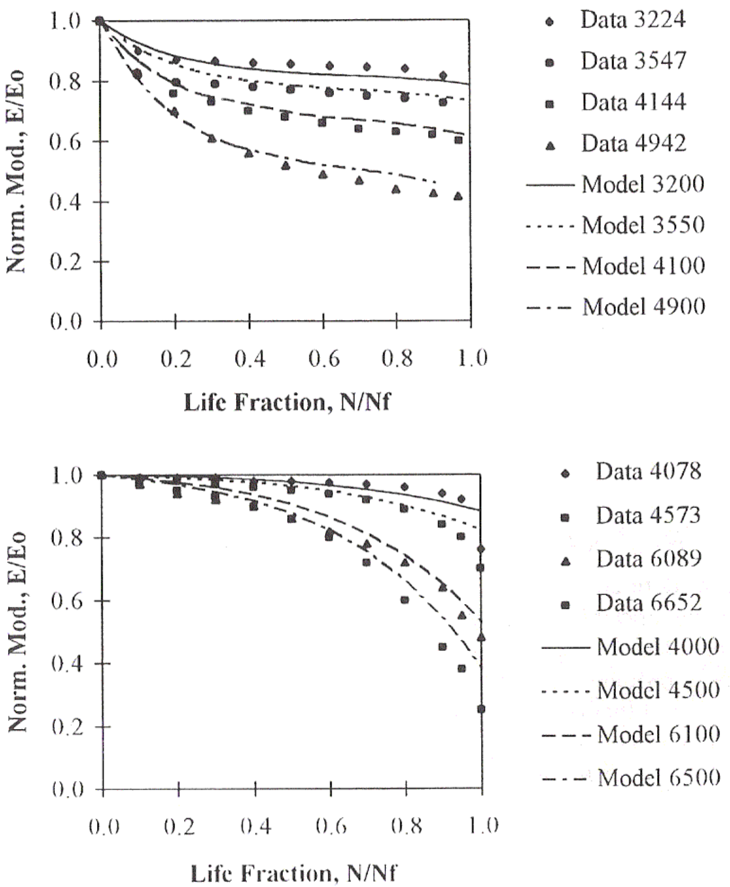


FIGURE 5.12. Degradation of elastic modulus during the fatigue life for human femoral bone specimens loaded in tension *(upper graph)* and compression *(lower graph)*. Symbols are experimental data from Pattin et al. (1996); curves are from the theory described in Section 5.7. In both cases, numbers shown in the legend represent initial strain values in με. (Reprinted from *J Biomechanics,* Vol. 29, Pattin, CA, Caler, WE, Carter, DR, Cyclic mechanical property degradation during fatigue loading of cortical bone, 69–79, 1996, with kind permission from Elsevier Science, Ltd., The Boulevard, Langford Lane, Kidlington 0X5 1GB, UK.)

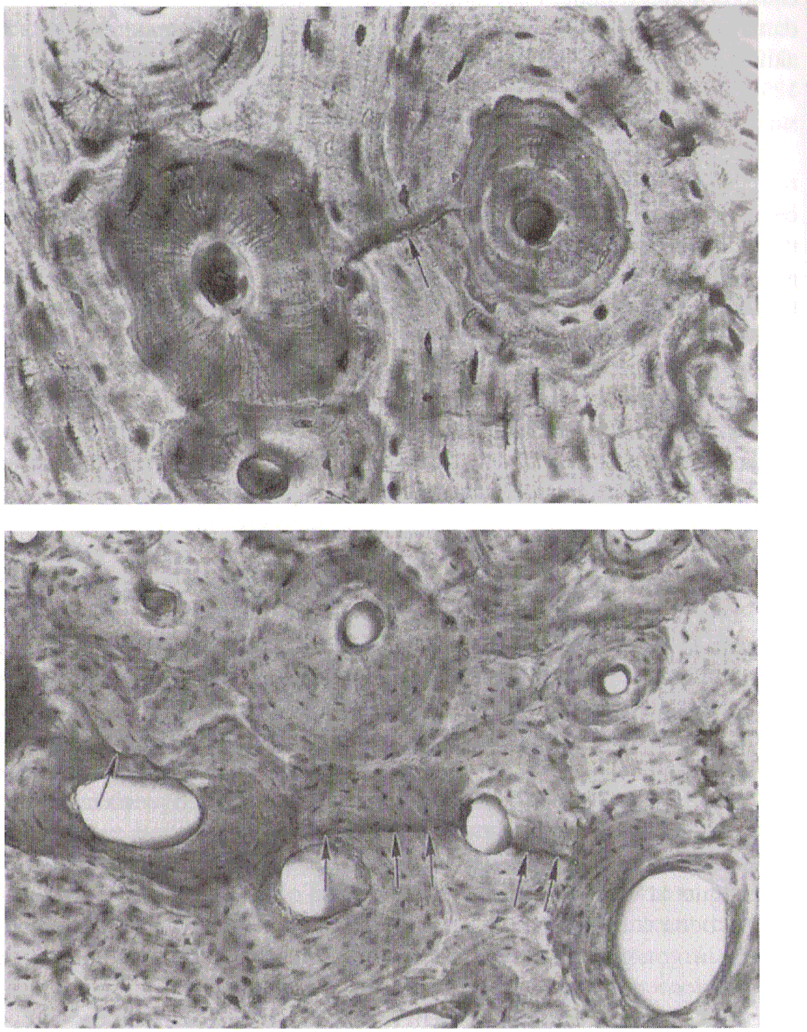
Evolution of the elastic modulus with the fatigue life:

• in tension the elastic modulus diminish rapidly, flowed by a prolonged much more gradual loss, and suddenly the modulus fall and failure occur.

• in compression the modulus falls slowly at first but the rate increases until failure.

• the fatigue life is longer in compression than in tension.

Fatigue behavior of bone – compression vs tension



A

B

FIGURE 5.17A,B. Microdamage (indicated by *arrows*) typical of (**A**) tensile and (**B**) compressive regions in a beam loaded in four-point bending. Human femoral cortex, 100-μm-thick section stained with basic fuchsin. Field width, ~500 μm. (Reproduced with permission from Griffin et al., 1997.)

• also the type of cracks is different in tension and in compression.

• in tension the cracks are placed in the interstitial space of the cortical bone.

•In compression crack tends to go from a Havers canal to another Havers canal .

Fatigue and bone remodeling

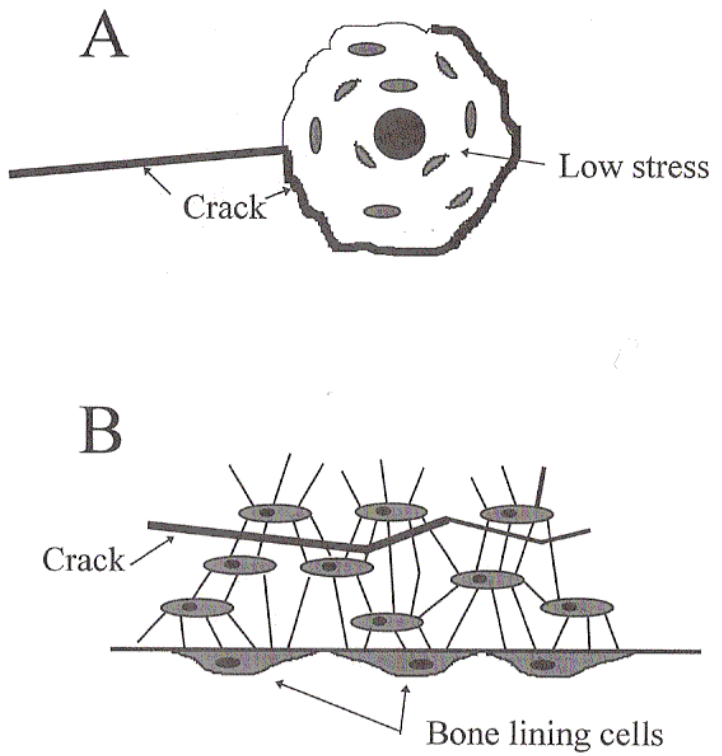


FIGURE 5.19A,B. Alternative hypotheses regarding the mechanism for activation of remodeling by fatigue damage. **A** debonding crack in cement line isolates osteon from stress, creating local disuse state that produces activation signal. **B** Damage to osteocytic network produces activation signal.

- the main function of remodeling is to renew bone, namely removing the fatigue damage.
- It has been estimated that the bone turnover and the fatigue live are related, i.e, the period of bone turn over coincide with the estimated fatigue life.
- there are theories saying that cracks break the osteocytic network, inhibiting the communication between osteocytes and originate the process of bone remodeling.
- several studies conclude that there an intense remodeling activity when the quantity of cracks is big.

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