Bone Tissue Mechanics

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PART 5

Material Properties of Cortical Bone

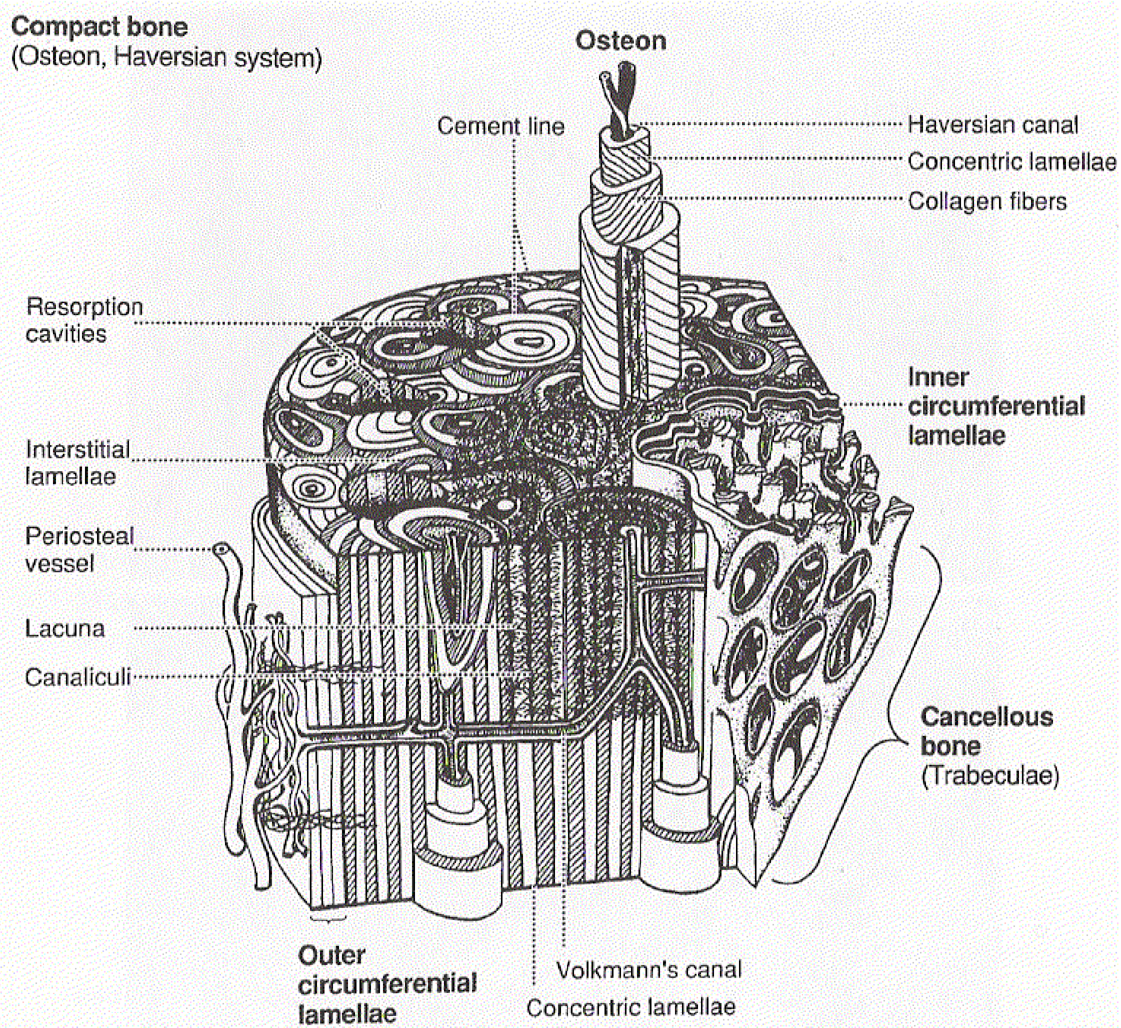


FIGURE 1.5 Diagram of a portion of a long bone shaft containing histological details of cortical bone. (From Weiss, L., Ed., *Cell and Tissue Biology, A Textbook of Histology,* Urban and Schwarzenberg, Baltimore, 1988. With permission.)

- The secondary osteons result from remodeling
- Osteons affect the mechanical properties of cortical bone:
  - By the replacement of highly mineralized bone matrix with less calcified material
  - Increasing the porosity
  - Altering the collagen fibers orientation
  - Introduction of a cement line interface.

Ø *osteons* = **200** ~ 300 µm
Ø canal = **50** ~ 90 µm
Ø vessel = 15 µm
$Ø_{max}$ lacuna = 10 ~ 20 µm

Cortical bone – secondary *osteons*

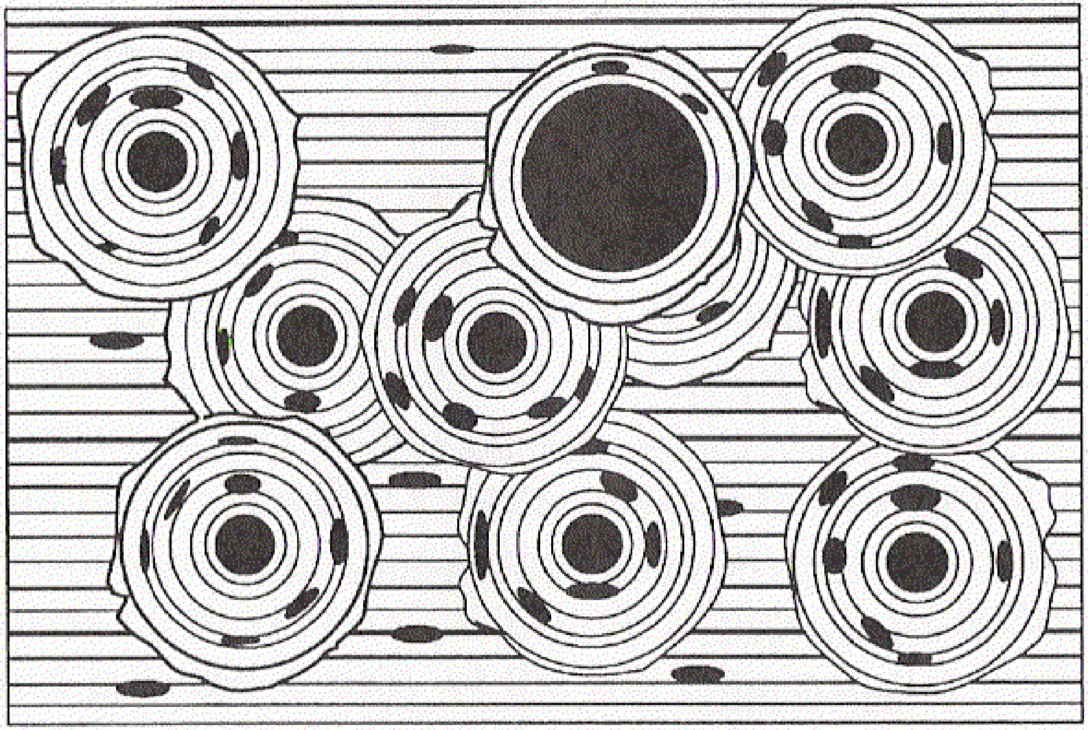


FIGURE 2.8. Schematic diagram of secondary osteons on a field of primary bone. One osteon is still forming; it has overlapped the Haversian canal of an existing osteon.

Mechanical properties of *osteons* – tension test

Ascenzi and Bonucci

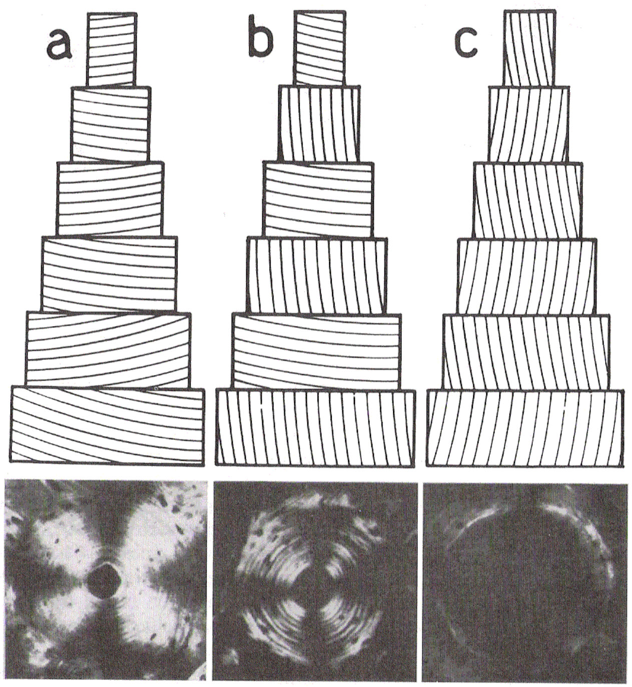


FIGURE 2.5. Three osteon types as defined by Ascenzi and Bonucci. Photomicrographs at *bottom* show appearance in plane-polarized light; diagrams *above* show hypothesized fiber arrangements in successive lamellae. *a*, Type T or transverse (i.e., circumferentially wrapped) fiber orientatio; *b*, type A or alternating fiber orientations; *c*, type L or longitudinal fiber orientation. (Reproduced with permission from Ascenzi and Bonucci, 1967.)

a = type T (transverse)
b = type A (alternating)
c = type L (longitudinal)

Tension test of osteonal wall segment

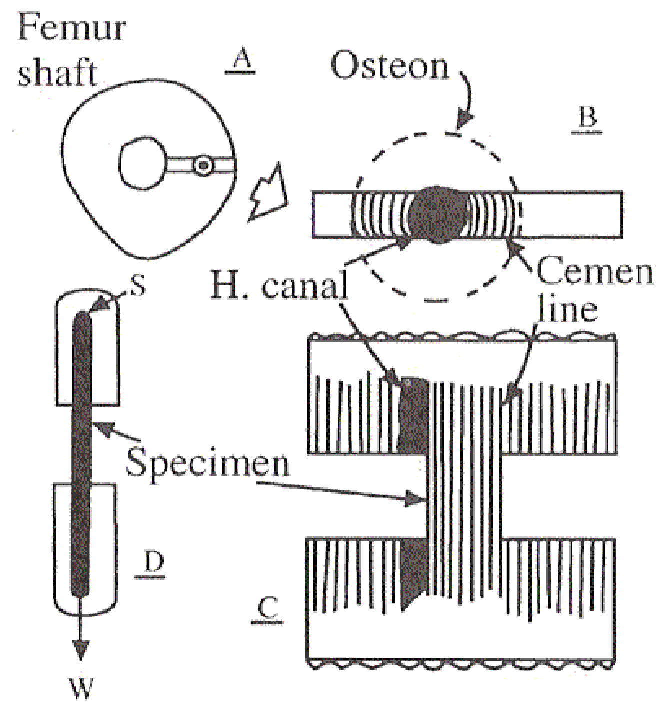
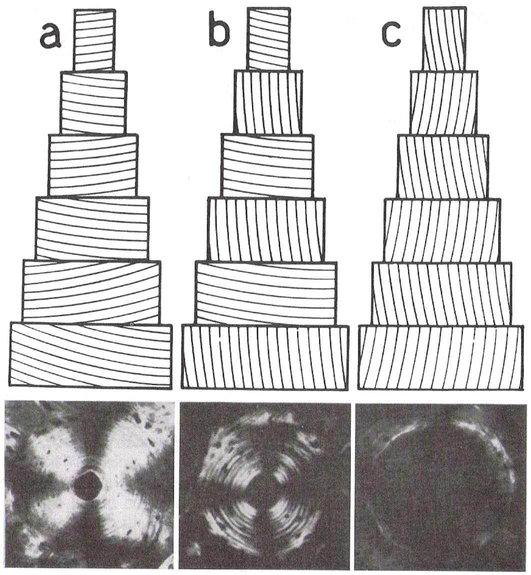


FIGURE 4.7. Diagram showing tension test of osteonal wall segment. *A*, End view of test slab removed from femur cross section. *B*, Relationship of a single osteon's geometry to the slab in an enlarged end view. *C*, Side view of central portion of the test slab. Material has been removed to leave only the lamellae on the right side of the osteon in place. *D*, The test slab supported at *S* and pulled in tension by weight *W*. (Drawn from description by Ascenzi and Bonucci, 1964, 1967.)

Mechanical properties of *osteons* – tension test

a = type T , b = type A , c = type L



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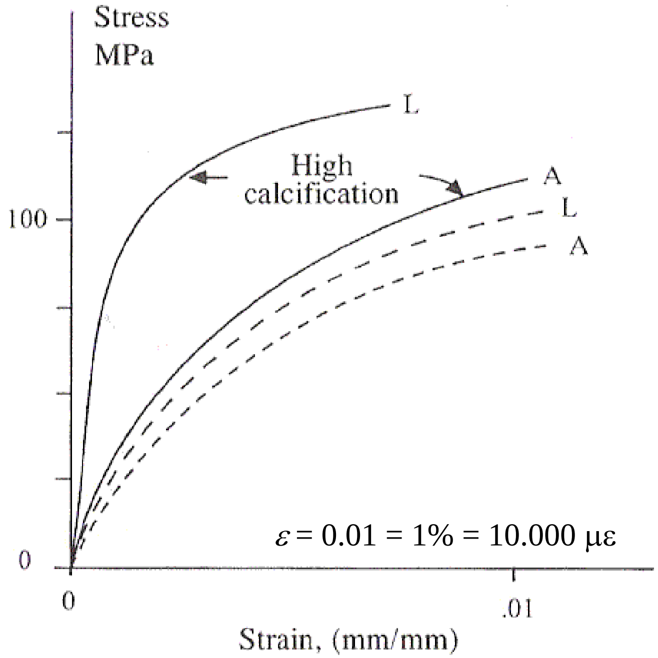

$\varepsilon = 0.01 = 1\% = 10.000\ \mu\varepsilon$

FIGURE 4.8. Typical stress-strain curves for osteon segments in tension. *Solid curves*, fully calcified osteons; *dashed curves*, osteons in the initial stages of calcification. *L*, type L osteon; *A*, type A osteon (see Fig. 2.5). (Redrawn from The tensile properties of single osteons, Ascenzi, A, Bonucci, E, *Anatomical Record*, 1967. With permission of Wiley-Liss, Inc., a subsidiary of John Wiley & Sons, Inc.)

• type L osteon have greater tensile strength and lower ultimate deformation than the type A osteons.

• the degree of mineralization has little effect on type A osteons

•the degree of mineralization has great effect on the type L osteons, both on stiffness and deformation until rupture.

• with less mineralization the two types of osteons has similar behavior.

Mechanical properties of *osteons*

a = type T , b = type A , c = type L

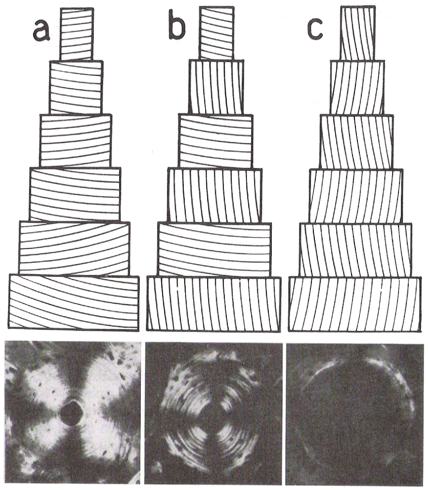

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TABLE 4.2. Mechanical properties of different types of osteons

Mode of loading	Ultimate stress, MPa	Elastic modulus, GPa	Ultimate strain, %
Tension			
Type L	114 ± 17	11.7 ± 5.8	6.8 ± 2.9
Type A	94 ± 15	5.5 ± 2.6	10.3 ± 4.0
Type T	—	—	—
Compression			
Type L	110 ± 10	6.3 ± 1.8	2.5 ± 0.4
Type A	134 ± 9	7.4 ± 1.6	2.1 ± 0.5
Type T	164 ± 12	9.3 ± 1.6	1.9 ± 0.3
Shear			
Type L	46 ± 7	3.3 ± 0.5[a]	4.9 ± 1.1[b]
Type A	55 ± 3	4.1 ± 0.4[a]	4.6 ± 0.6[b]
Type T	57 ± 6	4.1 ± 0.4[a]	4.6 ± 0.6[b]

[a]The shear modulus is defined as the slope of the linear portion of the stress-strain curve.

[b]The strain in the shear test is defined as the ratio of punch advancement to section thickness, expressed as a percent.

- under compression the type T osteons have greater strength and they are stiffer, while the type L osteons are less strength and less stiff.
- under shear the type A and type T osteons have similar properties, with greater strength and stiffness than type L osteons.
- the degree of mineralization increases the strength and stiffness.

Mechanical Properties - Anisotropy of Cortical Bone

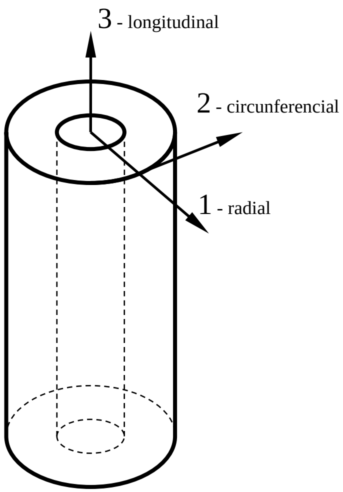
TABLE 4.4. Anisotropy of bovine and human bone

Elastic Modulus, GPa	Tension Longitudinal	Tension Transverse	Compression Longitudinal	Compression Transverse
Human				
Haversian	17.9 ± 0.9	10.1 ± 2.4	18.2 ± 0.9	11.7 ± 1.0
Bovine				
Haversian	23.1 ± 3.2	10.4 ± 1.6	22.3 ± 4.6	10.1 ± 1.8
Primary	26.5 ± 5.4	11.0 ± 0.2	—	—

Ultimate stress, MPa	Tension Longitudinal	Tension Transverse	Compression Longitudinal	Compression Transverse
Human				
Haversian	135 ± 16	53 ± 11	105 ± 17	131 ± 21
Bovine				
Haversian	150 ± 11[a]	49 ± 7[a]	272 ± 3	146 ± 32
Primary	167 ± 9	55 ± 9[a]	—	—

[a]Standard deviations approximate because groups were averaged.
From Reilly et al., 1974; Reilly and Burstein, 1975.

- The stiffness and strength for the human and bovine secondary bone are similar in the transverse direction but in the longitudinal direction the values are higher for bovine bone.
- The anisotropy ratios are less in human Haversian bone than in bovine Havarsian bone.
- The ultimate stress is very different for compression and tension



- comparing the ultimate stress in the longitudinal, circumferential and radial directions for bovine bone the following ration were obtained:
    - primary bone → 3 : 1 : 0.4
    - secondary bone (tension) → 3 : 1 : 0.7
    - secondary bone (compression) → 3 : 1 : 1
- osteons make whole bone transversely isotropic with respect to strength

Mechanical properties – elastic properties of bovine bone

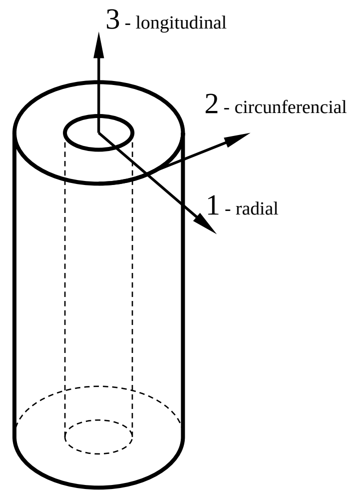
• valores obtidos por medições ultrasónicas

TABLE 4.5. Elastic coefficients in GPa

	Plexiform	Haversian
C_{11}	22.4 ± 0.6	21.2 ± 0.5
C_{22}	25.0 ± 1.0	21.0 ± 1.4
C_{33}	35.0 ± 2.0	29.0 ± 1.0
C_{44}	8.2 ± 0.4	6.3 ± 0.4
C_{55}	7.1 ± 0.3	6.3 ± 0.2
C_{66}	6.1 ± 0.2	5.4 ± 0.2
C_{12}	14.8 ± 0.8	11.7 ± 0.7
C_{23}	13.6 ± 0.7	11.1 ± 0.8
C_{13}	15.8 ± 0.8	12.7 ± 0.8

The C_{ij} subscripts refer to the following directions: 1 = radial direction; 2 = circumferential direction; 3 = longitudinal direction; 4 = circumferential-longitudinal shear; 5 = radial-longitudinal shear; 6 = radial-circumferential shear.

From Katz et al., 1984.



transversalmente isotrópico

$$\boldsymbol{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ & C_{11} & C_{13} & 0 & 0 & 0 \\ & & C_{33} & 0 & 0 & 0 \\ & & & C_{44} & 0 & 0 \\ & & & & C_{44} & 0 \\ \text{sim.} & & & & & \frac{1}{2}(C_{11}-C_{12}) \end{bmatrix}$$

• Secondary bone present a transversely isotropic behavior

• Due to bone remodeling (haversian bone) the behavior goes from orthotropic to transversely isotropic

Cortical Bone – influence of porosity

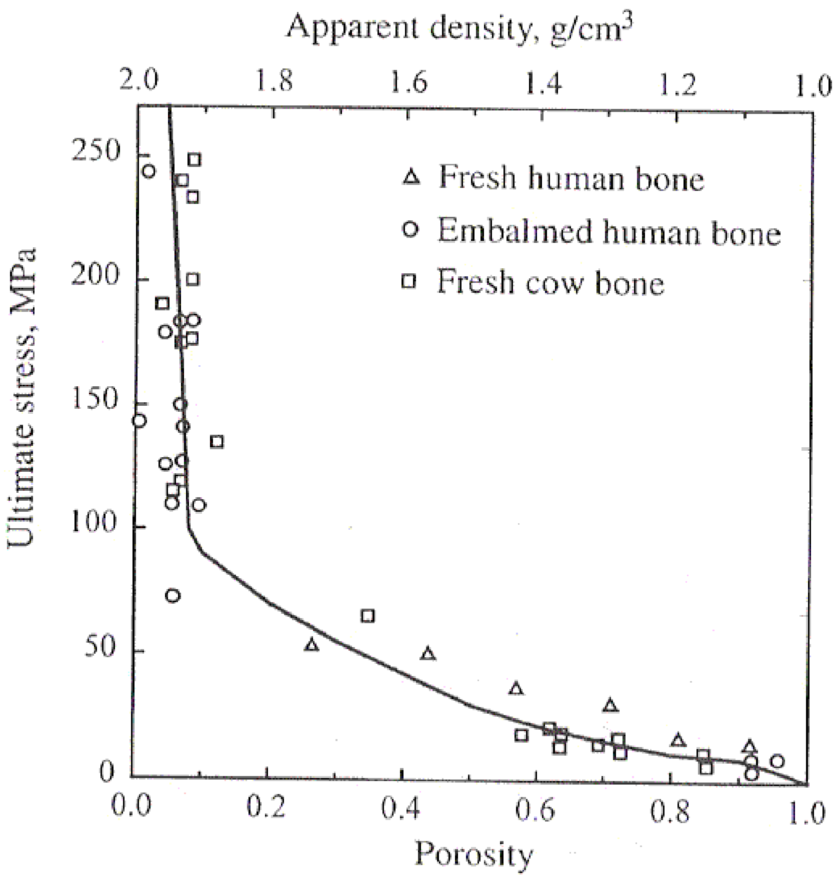


FIGURE 4.11. The relationships between ultimate compressive stress, porosity, and apparent density for fresh human bone *(triangles)* (Behrens et al., 1974), embalmed human bone *(circles)*, and fresh bovine bone *(squares)* (Martin, 1984). Very small porosity increases greatly affect cortical bone strength. (Replotted from data in Martin, 1984.)

Elastic Modulus:

From diverse studies (Schaffler and Burr, 1988, Carter and Hayes, 1977, Rice *et al.*, 1988) we can consider that the elastic modulus is proportional to a power of the volume fraction:

Cortical bone→ $E \sim (1-p)^{7.4}$

Trabecular bone→ $E \sim (1-p)^{2}$
Cortical and trabecular bone→ $E \sim (1-p)^{3}$

Cortical bone – influence of the collagen fibers orientation

Fiber orientation   mineralization

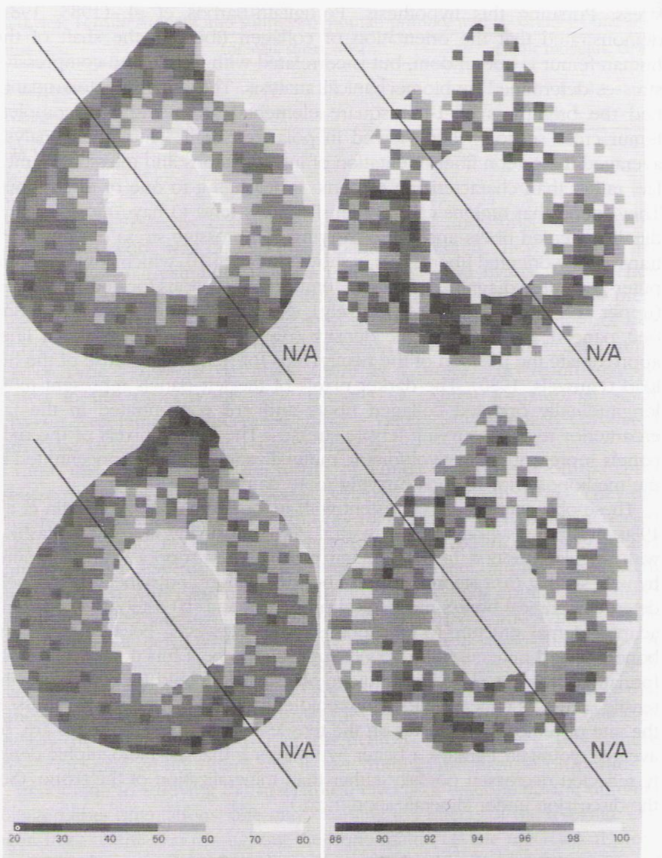


FIGURE 4.10. Plots of distribution of collagen fiber orientation *(left panels)* and radiographic density *(right panels)* in two cross sections from a human femur. These sections were located 1 cm apart in the central diaphysis. Superimposed on the plots are the approximate locations of the neutral axis (*N/A*) for bending produced by loading at the hip (Pauwels, 1980). The darker pixels of the left panels, which have more longitudinally oriented collagen fibers and greater tensile strength, are concentrated in the lateroanterior regions, carrying more tensile stresses. The lighter pixels of the *right panels,* representing greater volumetric mineralization, are more common in the medioposterior regions on the compression side of the neutral axis. Reproduced from Portigliatti-Barbos et al., 1983

• on the left panels, the darker pixels represent more fibers oriented in a longitudinal direction, and the lighter pixels represent more fibers oriented in the transversal direction.

•The darker region is where bone is under tensile stresses where the lighter correspond to bone in compression

•It suggest that collagen fibers tends to align depending on mechanical stimulus.

• The left panels show that the compression side is more mineralized.

Mechanical properties – cortical bone: rate of deformação, $\dot{\varepsilon}$

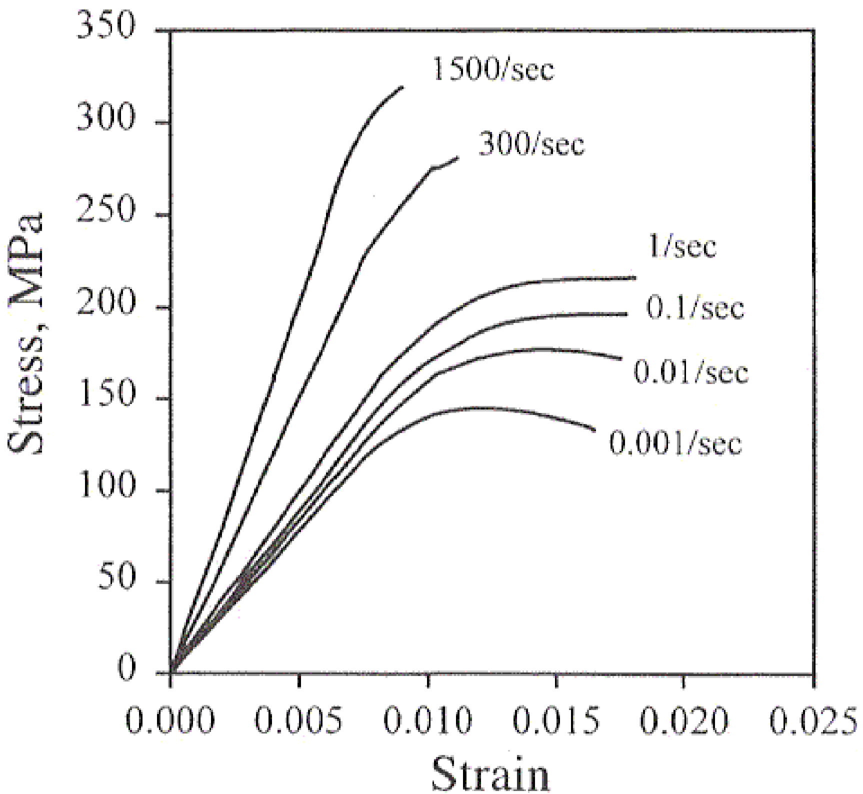


FIGURE 4.12. Stress-strain curves for cow bone loaded in compression at various strain rates. (Redrawn with permission from McElhaney, 1966.)

- Bone is a viscoelastic material, and thus, the properties depend on the rate of deformation.
- When the rate of deformation increases the bone stiffness and strength also increases, but the bone behavior is more fragile.
- The energy necessary to bone failure has a maximum for a rate of deformation of 0.01–0.10 s^{-1}

Mechanical properties of cortical bone - summary

In summary, the variables that have influence on properties of cortical bone are:

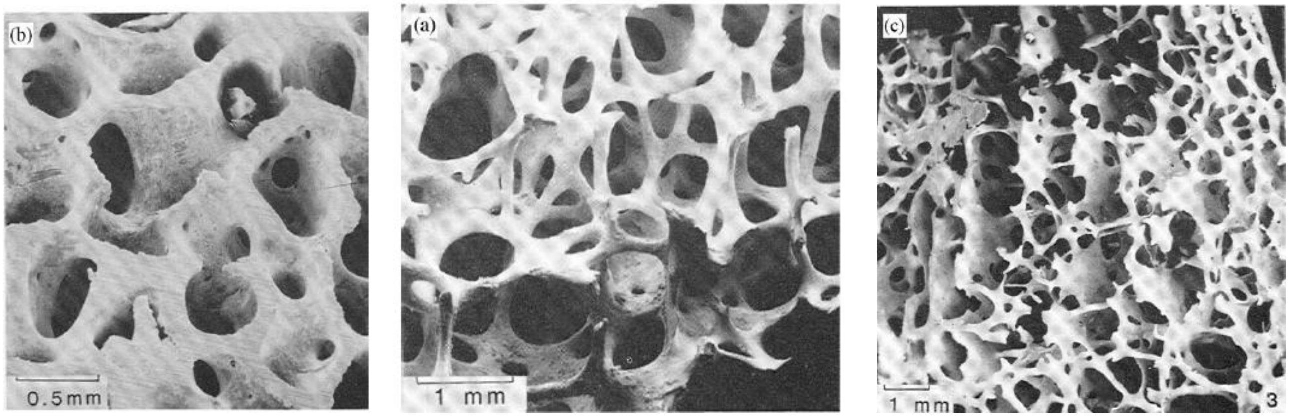
- •Porosity
- •Degree of mineralization
- •Orientation of collagen fibers

(the porosity and the degree of mineralization are the factors that define the bone apparent density).

These variables are affected by the histological bone structure (primary or secondary bone, lamellar vs woven bone, osteons, …).

Fatigue damage and rate of deformation are also import factors.

Mechanical Properties of trabecular bone – continous assumption



• trabecular thickness is about 100μm–300μm and the space between adjacent trabeculae is the order of 300μm–1500μm (these values don't not depend on animal size).

• to analyze bone in the context of continuum mechanics the samples have to have the minimum dimensions of 5mm–10mm (equivalent to 5 spaces between trabeculae).

• the study of trabecular bone for animals with small dimensions should consider the trabeculae as structures instead of a porous material in continuum mechanics

Mechanical Properties of trabecular bone

TABLE 4.1. Typical mechanical properties for cortical bone

Property	Human
Elastic modulus, GPa	
Longitudinal	17.4
Transverse	9.6
Tensile ultimate stress, MPa	
Longitudinal	133
Transverse	51
Compressive ultimate stress, MPa	
Longitudinal	195
Transverse	133

TABLE 4.7. Compressive strength and stiffness of cancellous bone

	Ultimate Stress, MPa	Modulus, MPa
Human		
Yamada, 1973[a]	1.86–1.37	90–70
Neil et al., 1983[b]	2.54 ± 0.62	272 ± 195
Kuhn et al., 1989a[c]	5.6 ± 3.8	424 ± 208
Rohl et al., 1991[d]	2.22 ± 1.42	489 ± 331
Canine		
Vahey et al., 1987[e]	12.1 ± 5.7	434 ± 174
Kuhn et al., 1989a[c]	7.12 ± 4.6	264 ± 132
Norrdin et al., 1990[f]	9.60 ± 0.80	231 ± 22

[a]Means for people in their forties and sixties, respectively; lumbar vertebrae.
[b]Lumbar vertebrae from men aged 54–90 years, 12-mm-diameter cylinders, 25–30 mm long.
[c]8-mm cubes from distal femur.
[d]Proximal tibia, 4 men and 3 women, aged 42–76 years.
[e]5-mm cubes from head and neck of the femur in 2 dogs.
[f]21 vertebral specimens from 7 dogs.

- <u>for a scale of cm</u>, trabecular bone is less stiff (more compliant) and less strength (weaker) than cortical bone.
- there is a great dispersion of values – the values are strongly influenced by porosity.
- The ultimate stress is equivalent under compression and under tension

Mechanical Properties of trabecular bone

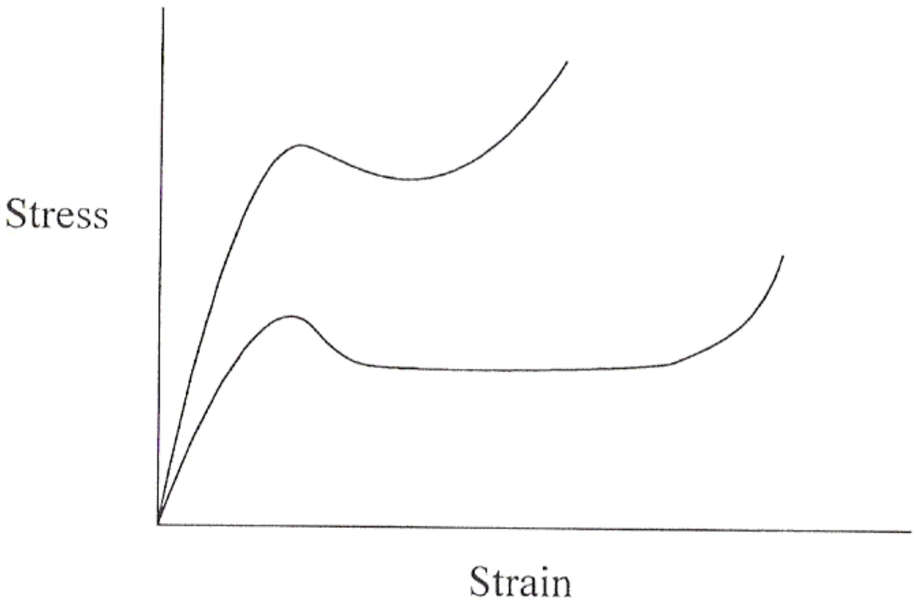
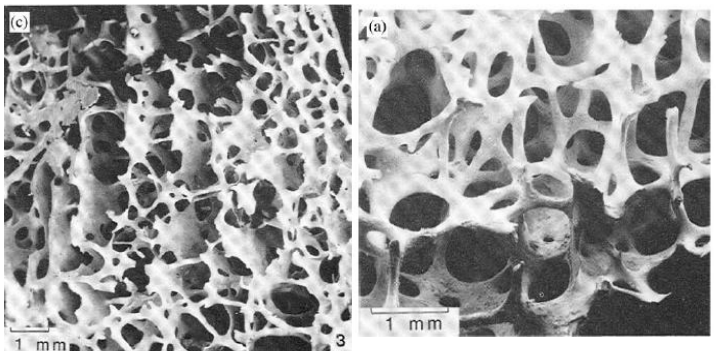
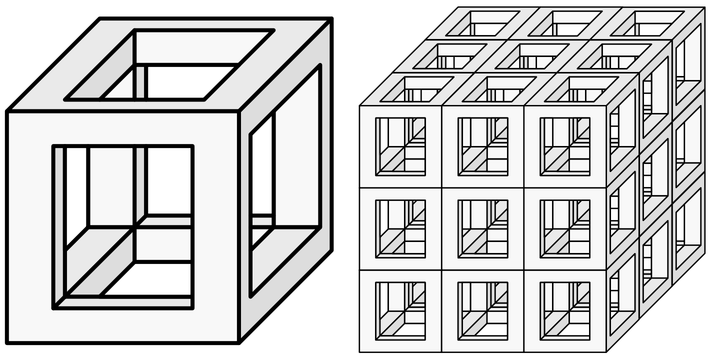


FIGURE 4.13. Load-deformation curves for two specimens of cancellous bone having different porosities. Yield and ultimate loads are apparent in the early portion of each curve. Subsequently, reduction and plateauing of the load occur. As void spaces collapse, the specimen densifies and the load rises steeply.





• <u>for a scale of cm</u> , the compression test for trabecular bone is very different of the compact bone.

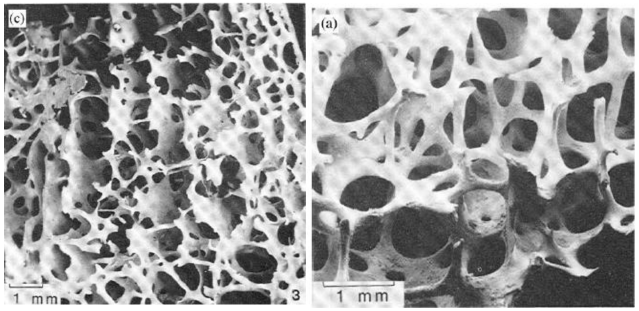
• a stress peak is followed by a reduction and than a plateau occurs. Finally the stress eventually increases. This behavior is consequence of the collapse and densification of trabeculae.

Mechanical Properties of trabecular bone

• elastic properties for a tibia (Ashman *et al.*, 1989) (the standard deviation is between parentheses)

Modulus	Mean Value (MPa)
E_1	346.8 (218)
E_2	457.2 (282)
E_3	1107.1 (634)
G_{12}	98.3 (66.4)
G_{13}	132.6 (78.1)
G_{23}	165.3 (94.4)

•1,2,3 are directions of orthotropy
1 = anterior-posterior
2 = medial-lateral
3 = inferior-superior



• note the high values for the standard deviation .

• note the degree of anisotropy, far from transversely isotropy.

Mechanical Properties of trabecular bone

The variables that influence the mechanical properties of trabecular bone are:

- porosity (or apparent density)
- trabeculae orientation
- the properties of an individual trabecula

remark:

- the third variable is less influent then the others two.
- the apparent density depend on porosity and degree of mineralization

Values for apparent density:

trabecular bone $\rightarrow \rho = 1.0 \sim 1.4$ g/cm^3

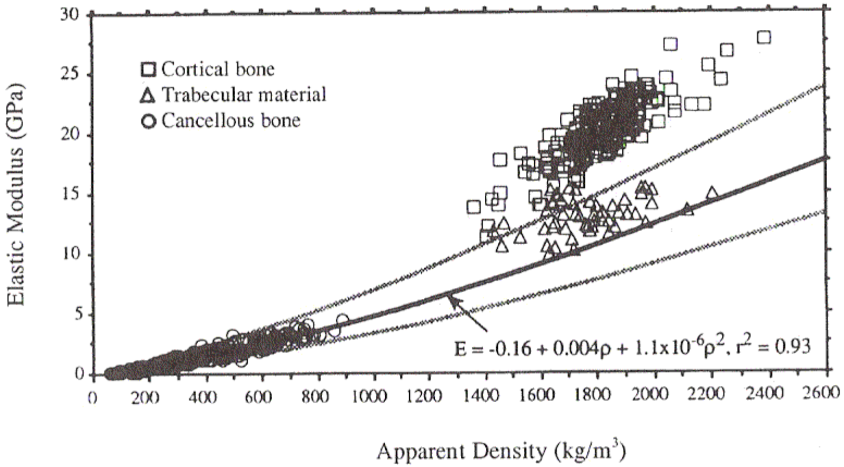
cortical bone $\rightarrow \rho = 1.8 \sim 2.0$ g/cm^3

remark:

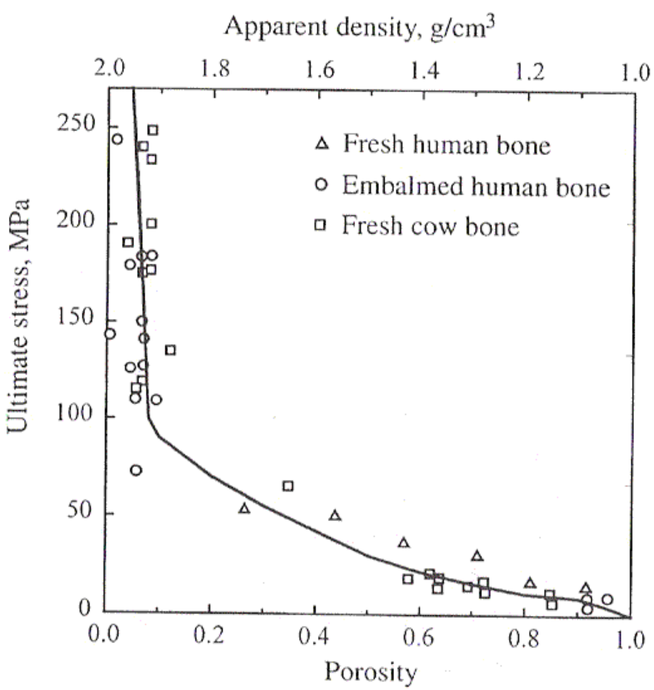
The apparent density, ρ, is measured considering the voids filled with soft tissue ($\rho = 1.0 \sim 2.0$ g/cm^3).

Some authors consider apparent density, d, considering empty voids ($d = 0.0 \sim 2.0$ g/cm^3).

Mechanical Properties of trabecular bone – influence of apparent density



• the influence of the apparent density on elastic modulus is clear in the graphs



• in literature the elastic modulus is referred as being proportional to a power (2 or 3) of the apparent density.

trabecular bone (only) $\rightarrow E \sim d^2$, $\sigma_U \sim d^2$
cortical and trabecular bone $\rightarrow E \sim d^3$

• apparent density (or porosity) is the parameter responsible for about 75% of the variability of mechanical properties of trabecular bone.

Mechanical Properties of trabecular bone – influence of the properties of trabecular tissue

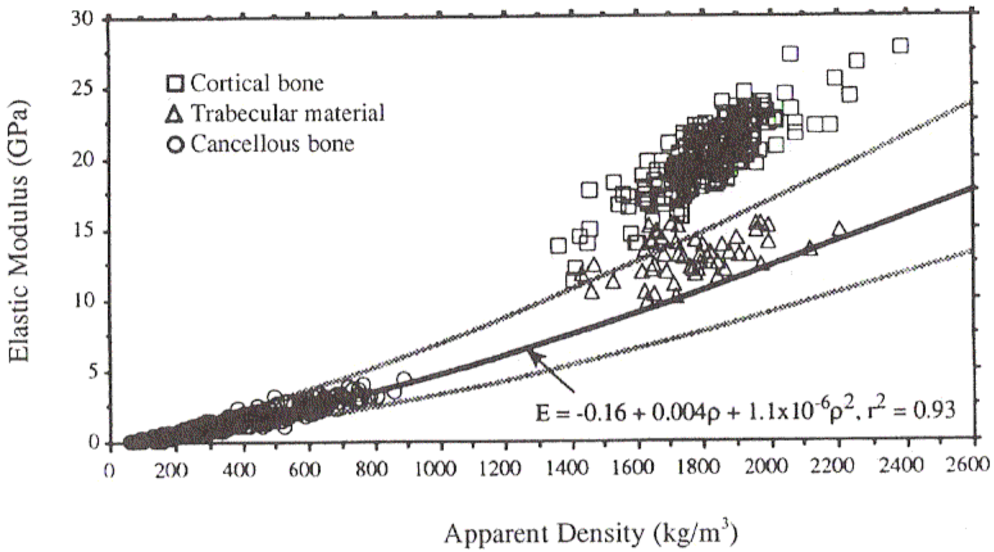


FIGURE 4.15. Ultrasonically determined elastic modulus vs. apparent density for continuum-level cancellous *(circles)* and cortical *(squares)* bone specimens. A linear regression line and its 95% confidence limits are shown for the cancellous specimens. The fact that the cortical specimens have modulus values above the upper confidence limit for the cancellous data indicates that the cortical bone tissue has a greater modulus than that of cancellous bone. When the tissue modulus of cancellous bone was measured and plotted against the apparent density of the tissue, the data *(triangles)* fell inside the confidence limits. (Reproduced from *Journal of Biomechanics*, Vol. 26, Rho et al., Young's modulus of trabecular and cortical bone material: ultrasonic and microtensile specimens, 111–119, 1993, with kind permission from Elsevier Science Ltd., The Boulevard, Langford Lane, Kidlington 0X5 1GB, UK.)

- other tests with samples of small dimension, show a decrease on bone properties.
- are the properties of trabecular tissue equivalent to the tissue of cortical bone?
- the bone of trabeculae is less stiff than the cortical tissue.
- it can be because of a lower degree of mineralization, but also because there is no intact fibers (osteons) inside a trabecula.

Mechanical Properties of trabecular bone – influence of the properties of trabecular tissue

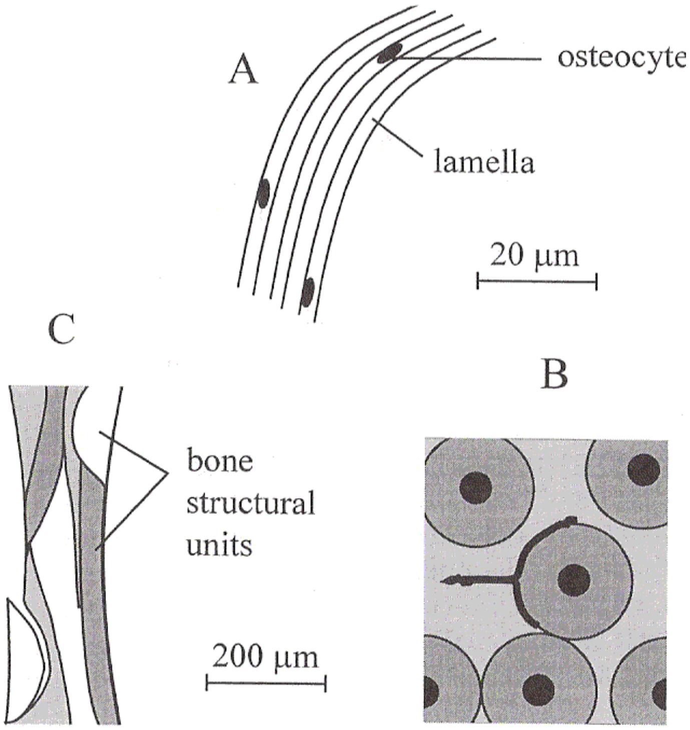


FIGURE 4.14. **A** At the level of lamellae, there is little to distinguish between osteonal and trabecular bone tissue. **B** In osteonal bone, the structural units produced by remodeling are largely intact, relatively long, fiberlike osteons. Their cement lines are internal to the bone; cement line disruptions are self-contained and do not compromise longitudinal loadbearing. **C** In trabeculae, the structural units produced by remodeling are much smaller and dish shaped with cement lines that, if disrupted, exfoliate the structural unit.

• are the properties of trabecular tissue equivalent to the tissue of cortical bone?

•The lamellar structure is identical.

•But due to the rate of remodeling the degree of mineralization is lower.

• there isn't entities like osteons totally integrated in a matrix. There is only part of these structures resulting of bone remodeling , where the boundary is not all inside the trabecula.

Mechanical Properties of trabecular bone – influence of trabecular orientation (trabecular architecture)

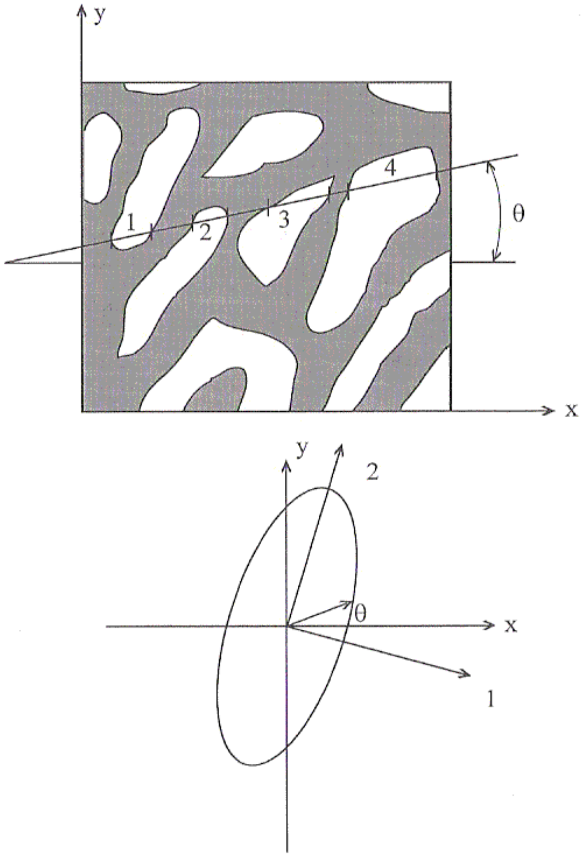


FIGURE 4.16. *Upper:* Schematic diagram of anisotropic cancellous bone structure as seen in a two-dimensional section also depicts an example of a test line used to measure mean intercept length. This line makes an angle Θ with the x-axis and the intercept lengths measured along it are shown as line segments *1–4*. *Lower:* Plot of mean intercept length (MIL) vs. Θ is an ellipse. The vector for test lines having orientation Θ is shown inside the ellipse. The ellipse has minor and major axes (labeled *1* and *2*) aligned with the principal directions of the two-dimensional version of the MIL tensor. Note that the maximum principal axis of the MIL ellipse is aligned with the general orientation of the trabeculae in the upper diagram.

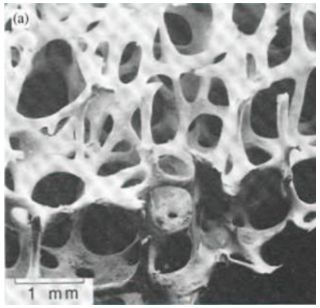
• the bone anisotropy depend on the trabecular orientation and on the space between trabeculae.

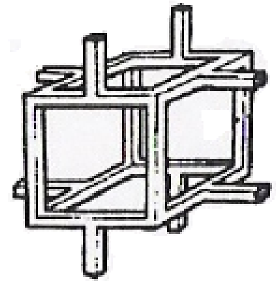
• one method to quantify the trabecular bone anisotropy is the MIL – *mean intercept length*

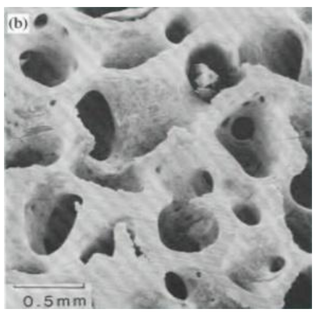
• Cowin (1985) derived the anisotropic bone properties based on bone apparent density and the f*abric tensor*, *H*, that is equal to the inverse square root of the MIL tensor.

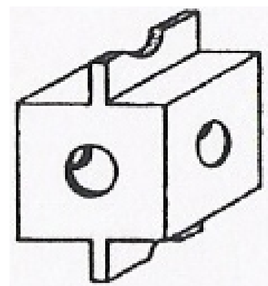
• the Cowin theory accounts for 72% – 94% of the variability in the elastic constants for cancellous bone.

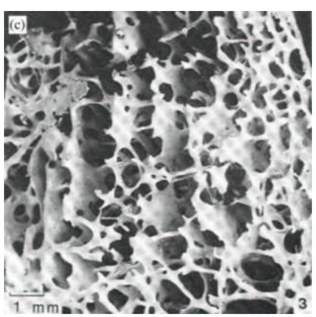
Modeling Cancellous Bone as a Cellular System of plates or Struts

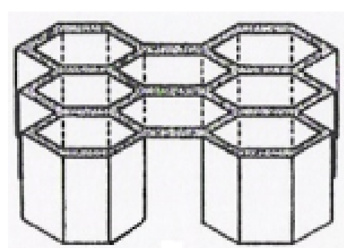


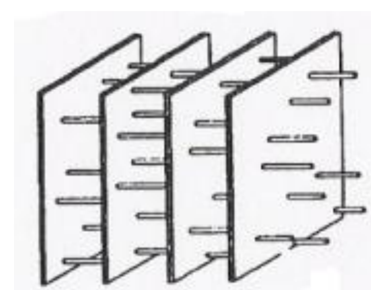












- Gibson (1985) derived relationships between equivalent material properties and porosity, using open and close cell porous structures.
- Results are consistent with previous ones, with stiffness varied as d^3 for closed cell (dense bone) and as d^2 for open cell (cancellous bone)

Predicting Material properties: Bone as a composite material

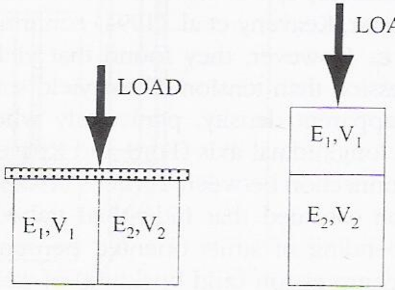


FIGURE 4.18. Voigt *(left)* and Reuss *(right)* models for composite material formed from a mixture of two materials. In the Viogt model, each material strains the same amount; in the Reuss model, each material carries the same stress.

- These models give upper and lower bonds for properties, considering bone a composite material made of two phases. For example, collagen and mineral, or soft and hard tissue.

Voigt $$P_c = P_1 F_1 + P_2 F_2$$

Reuss $$P_c = \frac{P_1 P_2}{P_1(1 - F_1) + P_2(1 - F_2)}$$

Mechanical properties of bone

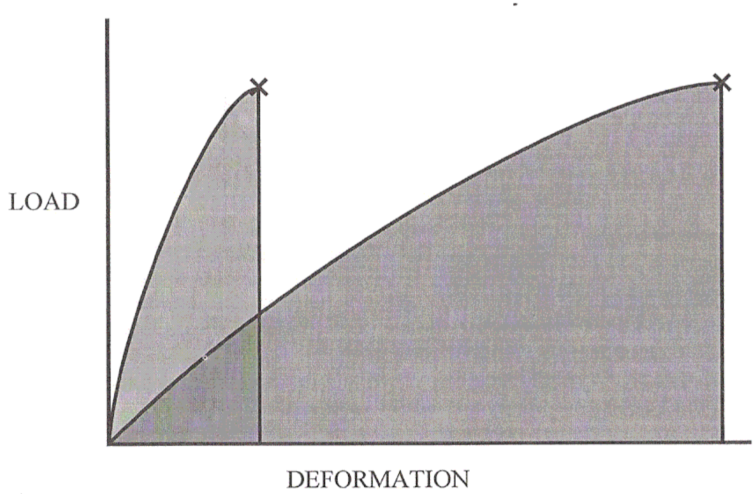


FIGURE 4.5. If two bones have the same strength (failure load), the more compliant one (at *right*) requires more energy to break it.

• <u>for an equivalent ultimate load</u>, a flexible material needs more energy to fail.

• a stiff bone makes the muscles action more efficient (muscle energy is not used to deform bone).
• a compliant bone is better to the protection function (it absorbs more energy before failure).
• a heavy bone is less efficient (less mobility)
• Bone structure is the result of all these compromises.

Fatigue

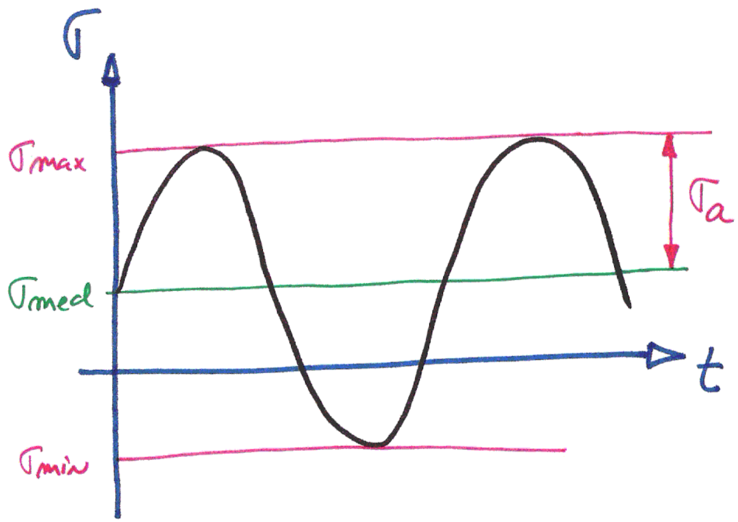
Cyclic loads

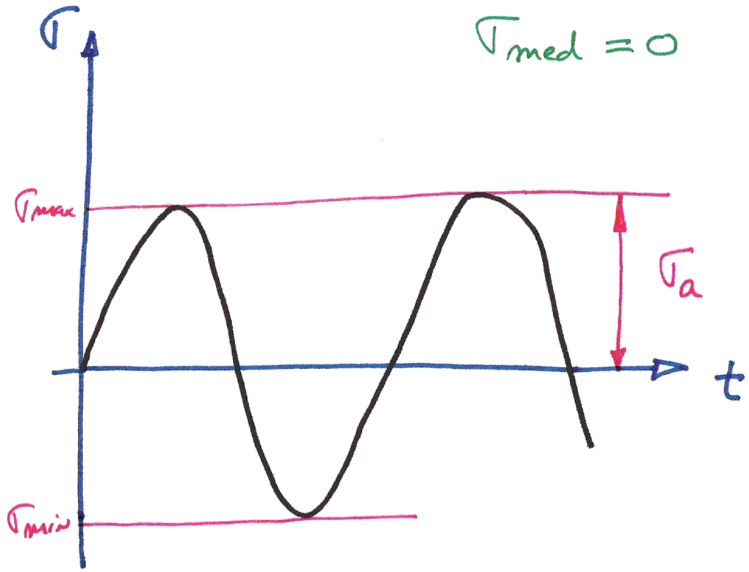
Fatigue – damage accumulation. The failure occurs even for load below the failure stress.

Fatigue Analysis

- low number of cycles ($< 10^3$; $\sigma > \sigma_y$)
- high number of cycles ($> 10^3$; $\sigma < \sigma_y$)
- fracture mechanics – crack propagation

Fatigue for a high number of cycles:



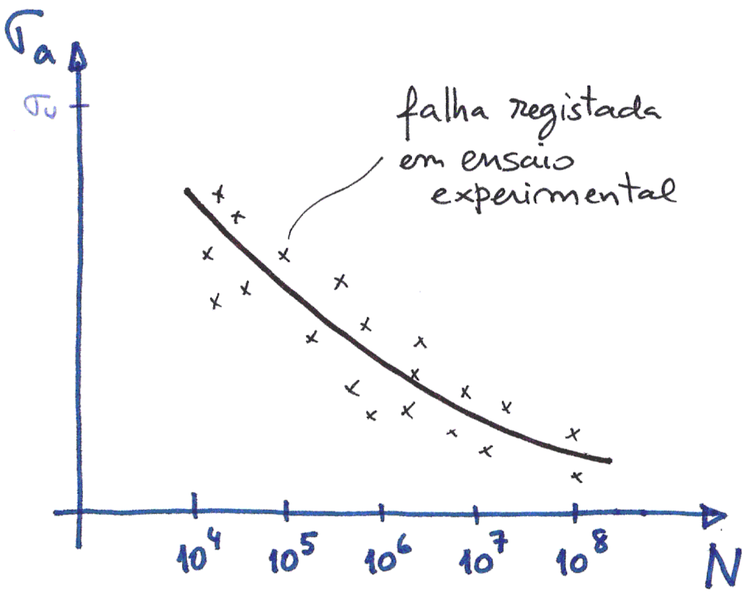


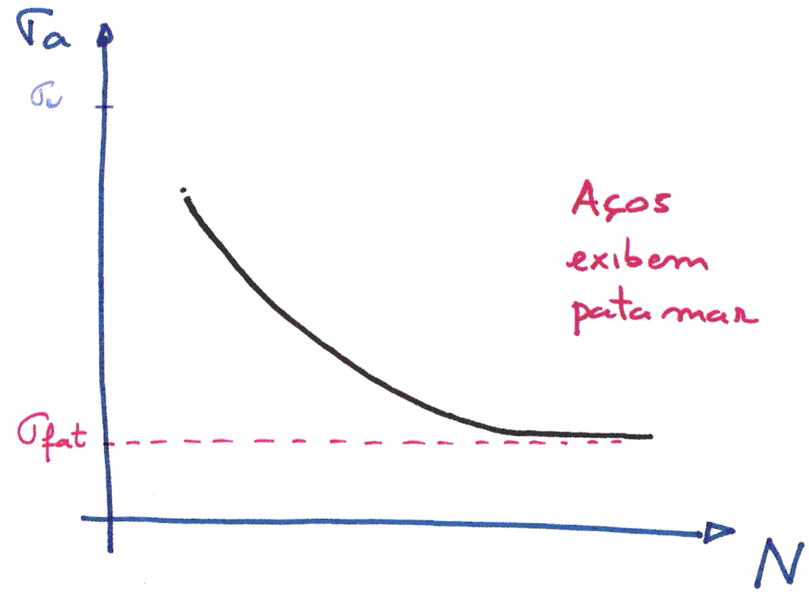
$$\sigma_{med} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

S-N curve (adjusted for experimental data):

When $\sigma_{med} = 0$





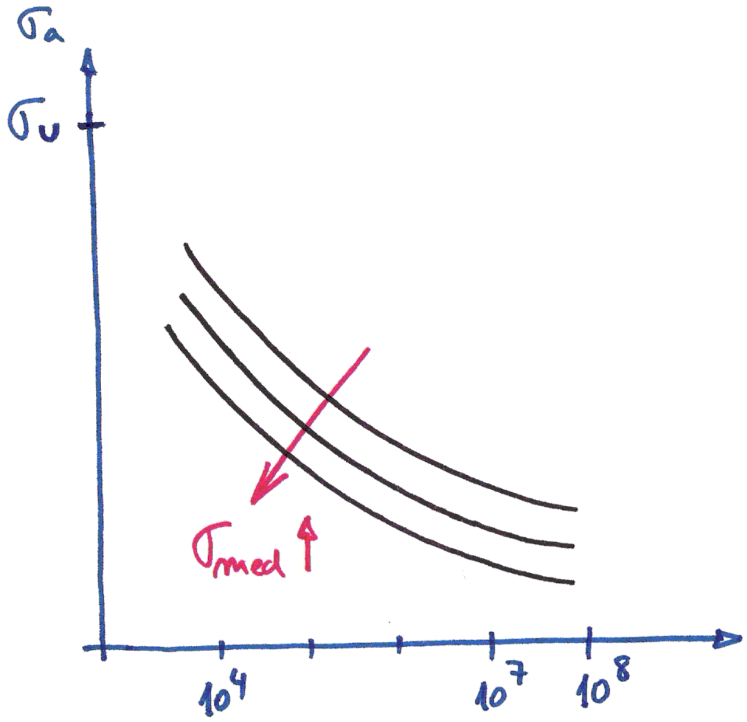
Curve of the type:

$$\sigma_a = \frac{b}{N^c} \Rightarrow N = \left(\frac{b}{\sigma_a}\right)^{1/c}$$

$$\sigma_a = \sigma_{fat} + \frac{b}{N^c}$$

$$\Rightarrow N = \left(\frac{b}{\sigma_a - \sigma_{fat}}\right)^{1/c}$$

Effect of σ_m



Alteration

Substituing σ_{fat} by σ'_{fat}

Goodman Relation (empirical)

$$\sigma'_{fat} = \sigma_{fat}\left(1 - \frac{\sigma_m}{\sigma_U}\right)$$

Problem

A beam is used to lift containers. Knowing that the weight of each container is P=356 kN and that the maximum bending moment is M_{max}=PL/8, determine the life-time for the beam if the beam works 200 days per year and it lifts 20 containers per day.

Beam properties

$h = 0,3175\ m$ (altura)

$I = 118,6 \times 10^{-6}\ m^4$

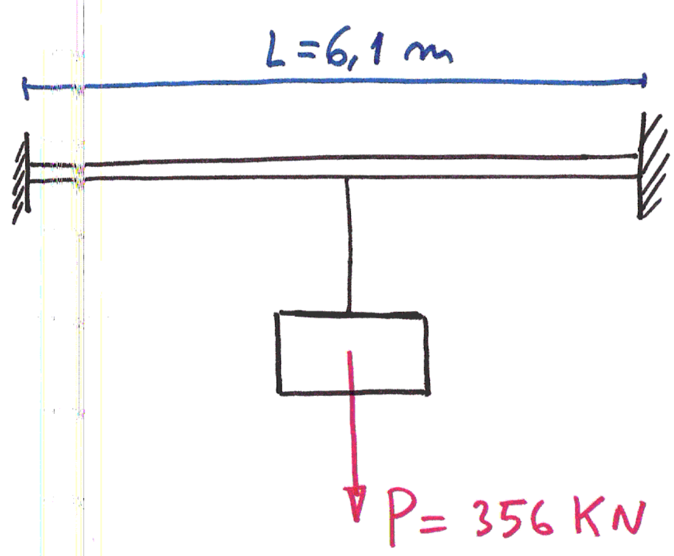
Material properties

$\sigma_u = 552\ MPa$

$\sigma_{fat} = 69\ MPa$

$b = 827\ MPa$

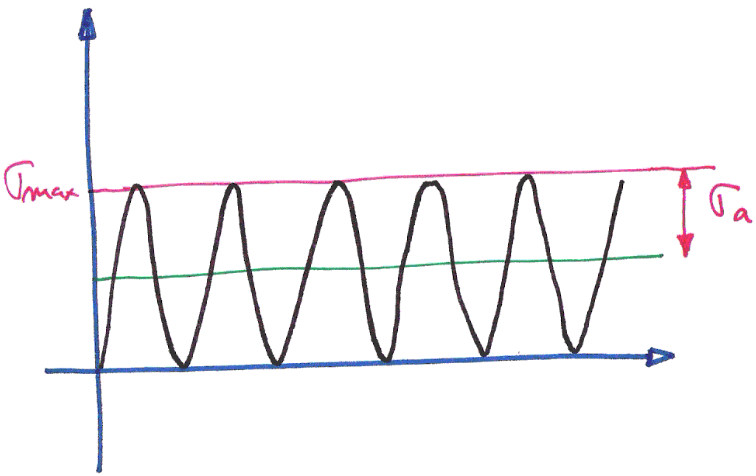
$c = 0,15$



Solution:

$$M_{max} = \frac{PL}{8} = \frac{356 \times 10^3 \times 6{,}1}{8} = 271{,}5 \text{ KNm}$$

$$\sigma_{max} = \frac{My}{I} = \frac{271{,}5 \times 10^3 \times 0{,}3175/2}{118{,}6 \times 10^{-6}} = 363 \text{ MPa}$$



$$\sigma_a = \frac{\sigma_{max} - \sigma_{med}}{2} = \frac{363 - 0}{2} = 181{,}5 \text{ MPa}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{med}}{2} = \frac{363 + 0}{2} = 181{,}5 \text{ MPa}$$

Goodman

$$\sigma_{fat} = \sigma_{fat}\left(1 - \frac{\sigma_m}{\sigma_u}\right) = 69 \times \left(1 - \frac{181{,}5}{552}\right) = 46{,}3 \text{ MPa}$$

Number of cycles:

$$N = \left(\frac{b}{\sigma_a - \sigma_{fat}'}\right)^{1/c} = \left(\frac{827}{181{,}5 - 46{,}3}\right)^{\frac{1}{0{,}15}} = 175 \times 10^3 \text{ ciclos}$$

Number of cycles/year:

20 cycles/day × 200 days/year = 4000 cycles/year

Duração

$$\frac{175.000 \text{ cycles}}{4.000 \text{ cycles/year}} = 43{,}75 \text{ years} \longrightarrow \text{about } 44 \text{ years}$$

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