Bone Tissue Mechanics

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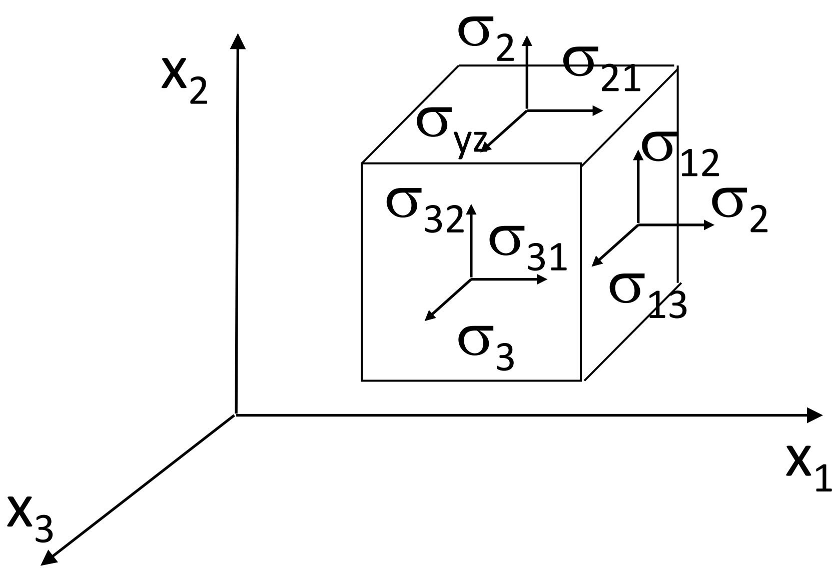
PART 2

Stress

$$\sigma_Q = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$

- Stress is a measure of the internal forces associated to the plane of interest.

- In general every plane containing the point Q has a normal and a shearing stress component.



- The general state of stress is described by the components in a x_1, x_2, x_3 reference system.

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

- Only six components because the tensor is symmetric.

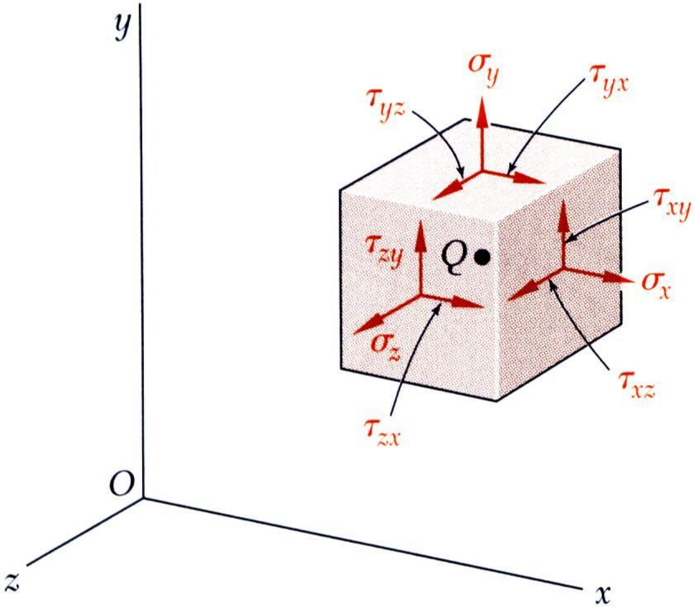
σ_{11}, σ_{22}, σ_{33} – normal stress

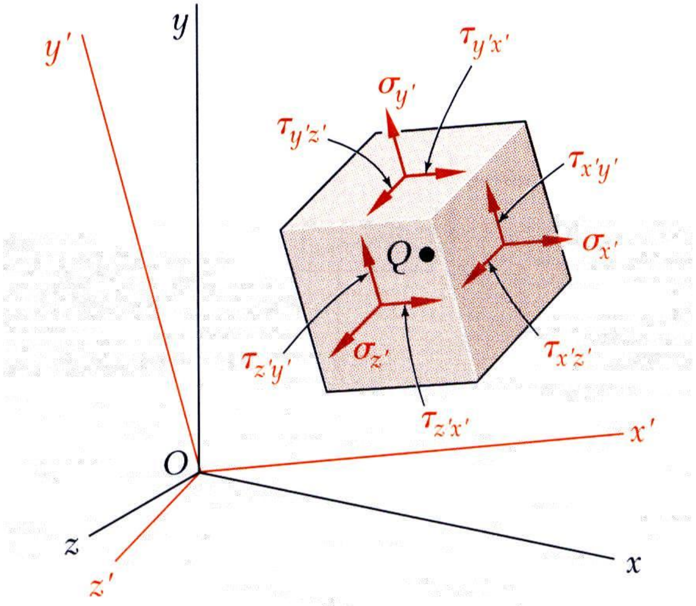
σ_{12}, σ_{13}, σ_{23} – shearing stress

Stress

- Stress components depend on the reference system.

- The same state of stress is represented by a different set of components if axes are rotated.





Beer & Johnston (McGraw Hill)

2D example

Transformation of coordinates: Problem 1

Assume the plane stress state given by its components in the x-y system (x is the horizontal axis and y is the vertical one):

$$[\sigma]=\begin{bmatrix} 6 & 3 \\ 3 & -2 \end{bmatrix}$$

Write the components of this stress tensor in the reference system which makes with the previous one:

a) 90º

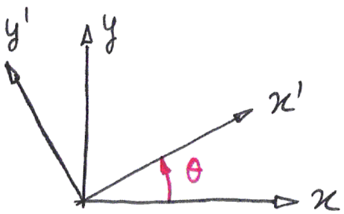
b) 18,4º

Transformation of coordinates

$$\sigma'_{ij} = R_{ik} \, R_{jl} \, \sigma_{kl}$$ (convenção de soma)

- σ'_{ij}: tensor escrito no referencial x'
- R_{ik}: matriz de rotação
- σ_{kl}: tensor escrito no referencial x

For an angle θ (and 2D)



$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix}$$

$$\sigma' = \,?$$

$$\sigma'_{ij} = R_{ik} R_{jl} \sigma_{kl} \Rightarrow [\sigma'] = [R][\sigma][R]^T$$

$$\boxed{\Rightarrow [\sigma'] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}$$

...

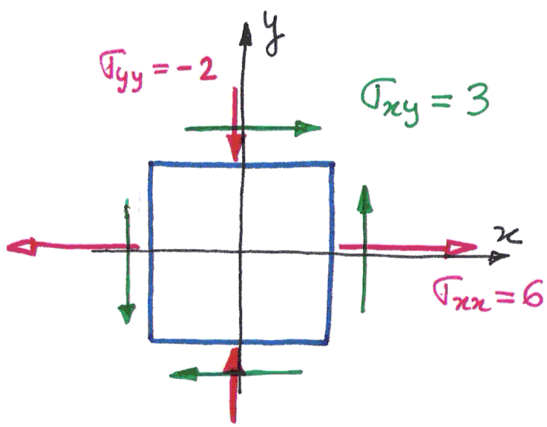
$$\Rightarrow [\sigma'] = \begin{bmatrix} \sigma_{xx}\cos^2\theta + 2\sigma_{xy}\cos\theta\sin\theta + \sigma_{yy}\sin^2\theta & (\sigma_{yy} - \sigma_{xx})\cos\theta\sin\theta + \sigma_{xy}(\cos^2\theta - \sin^2\theta) \\ \text{simétrica} & \sigma_{xx}\sin^2\theta - 2\sigma_{xy}\cos\theta\sin\theta + \sigma_{yy}\cos^2\theta \end{bmatrix}$$

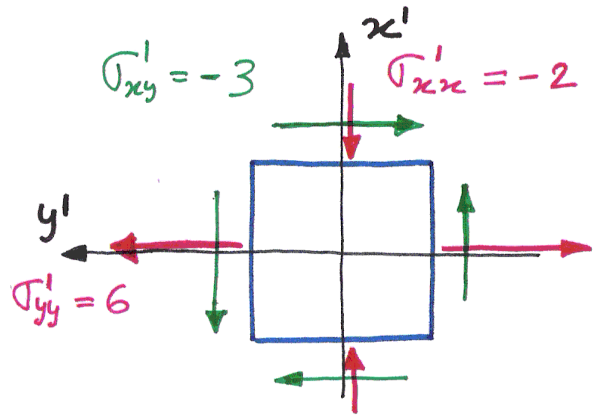
a) $[\sigma] = \begin{bmatrix} 6 & 3 \\ 3 & -2 \end{bmatrix}$ $\quad \underline{\underline{\theta = 90^\circ}} \Rightarrow [R] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$$[\sigma'] = [R][\sigma][R]^T \Rightarrow [\sigma'] = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 6 & 3 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow [\sigma'] = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 3 & -6 \\ -2 & -3 \end{bmatrix}$$

$$\Rightarrow [\sigma'] = \begin{bmatrix} -2 & -3 \\ -3 & 6 \end{bmatrix}$$



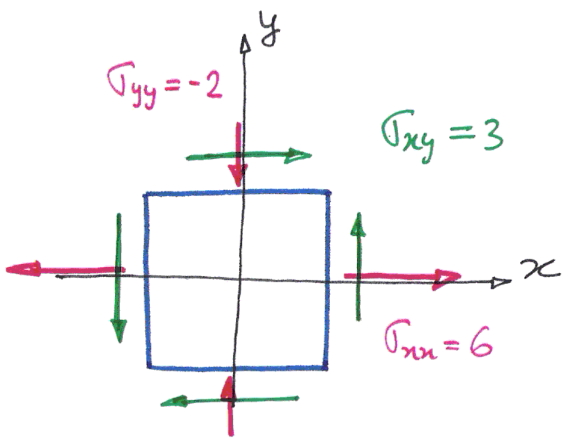


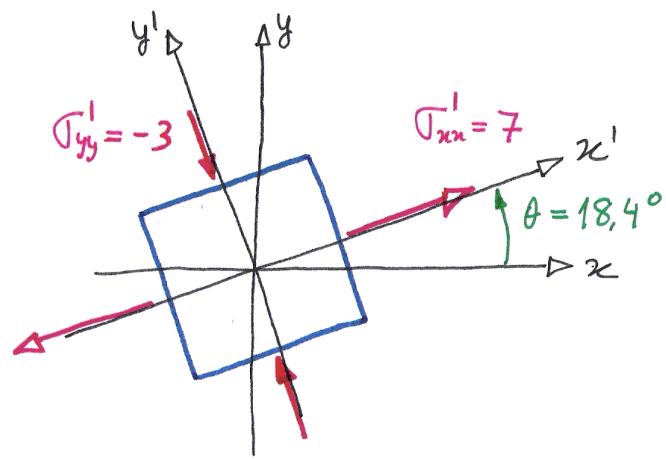
b) $[\sigma] = \begin{bmatrix} 6 & 3 \\ 3 & -2 \end{bmatrix}$ $\qquad \theta = 18{,}4^\circ \Rightarrow [R] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 0.9489 & 0.3156 \\ -0.3156 & 0.9489 \end{bmatrix}$

$$[\sigma'] = [R][\sigma][R]^T \Rightarrow [\sigma'] = \begin{bmatrix} 0.9489 & 0.3156 \\ -0.3156 & 0.9489 \end{bmatrix} \begin{bmatrix} 6 & 3 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 0.9489 & -0.3156 \\ 0.3156 & 0.9489 \end{bmatrix}$$

$$\Rightarrow [\sigma'] = \begin{bmatrix} 0.9489 & 0.3156 \\ -0.3156 & 0.9489 \end{bmatrix} \begin{bmatrix} 6.6402 & 0.9531 \\ 2.2155 & -2.8446 \end{bmatrix}$$

$$\Rightarrow [\sigma'] \cong \begin{bmatrix} 7 & 0 \\ 0 & -3 \end{bmatrix}$$





Transformation of coordinates: Problem 2 (Using the Mohr's Circle)

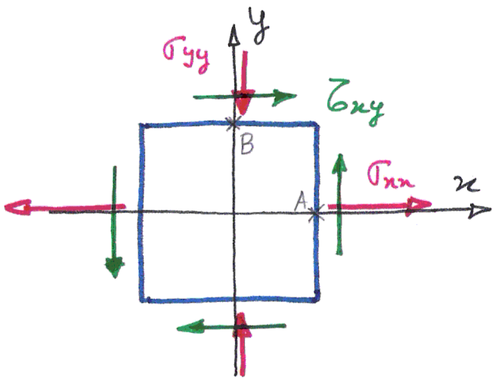
Assume the plane stress state given by its components in the x-y system (x is the horizontal axis and y is the vertical one):

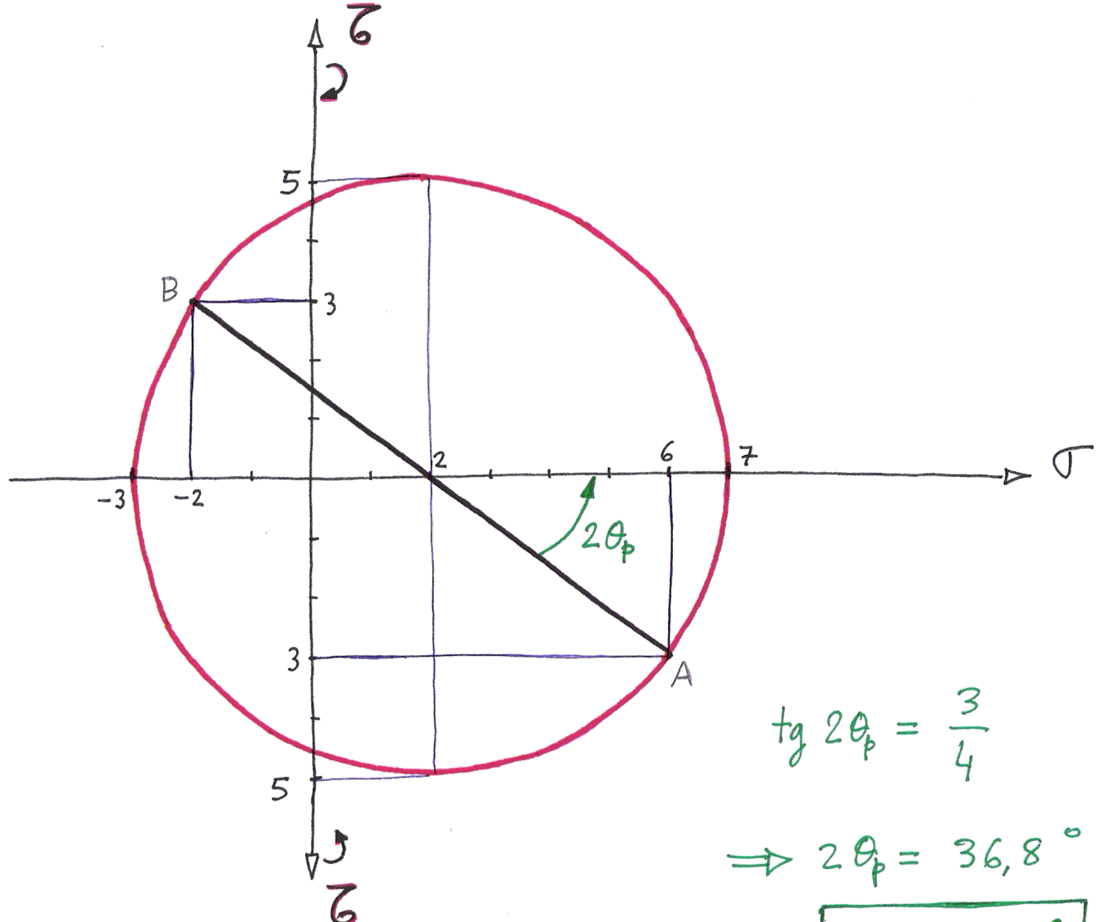
$$[\sigma]=\begin{bmatrix}6 & 3\\ 3 & -2\end{bmatrix}$$

Draw the Mohr's circle for this stress state.

Mohr's circle (2D)

$$[\sigma] = \begin{bmatrix} 6 & 3 \\ 3 & -2 \end{bmatrix}$$





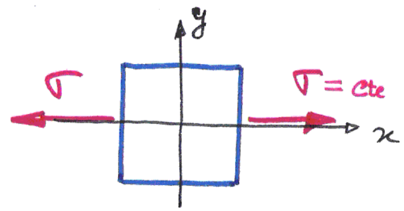
$$tg\ 2\theta_p = \frac{3}{4}$$

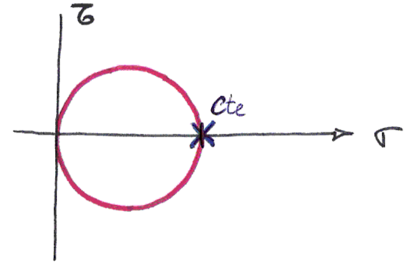
$$\Rightarrow 2\theta_p = 36,8^\circ$$

$$\Rightarrow \boxed{\theta_p = 18,4^\circ}$$

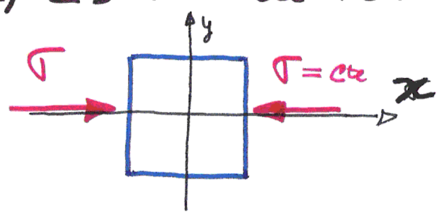
Notas

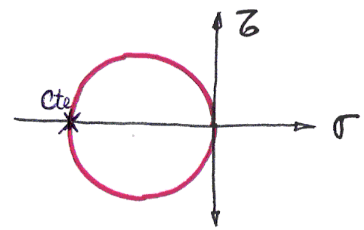
1) Estado de tensão de tracção uniaxial



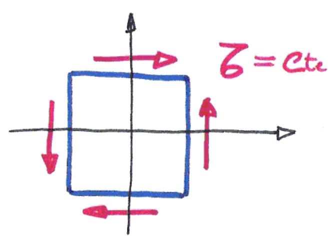


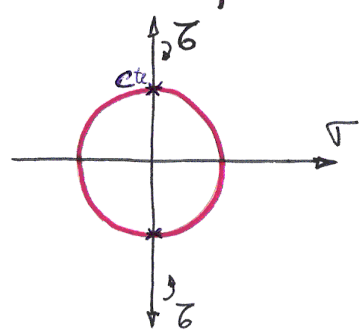
2) Estado de tensão de compressão uniaxial





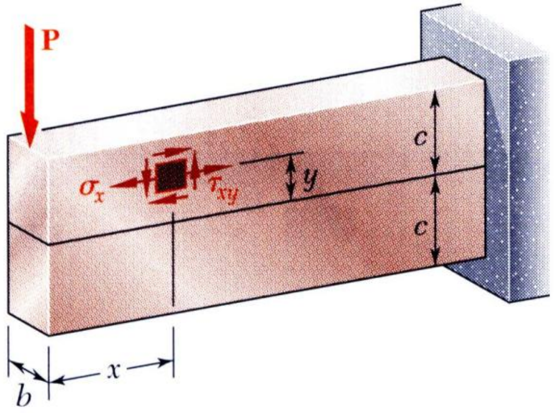
3) Estado de tensão de corte puro (torção)

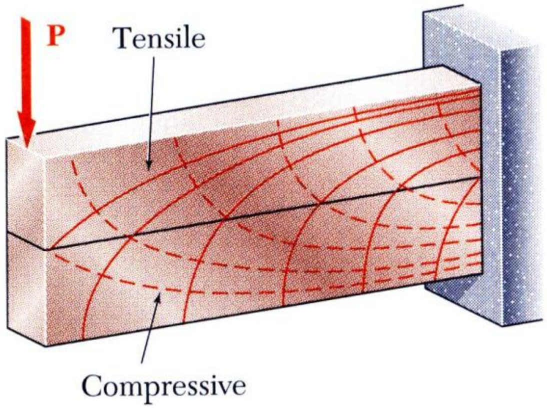




Principal Stresses

- Structures are often subject to different combined loads. For instance a beam is usually subject to normal stress due to bending and shear stress due to the transverse load.



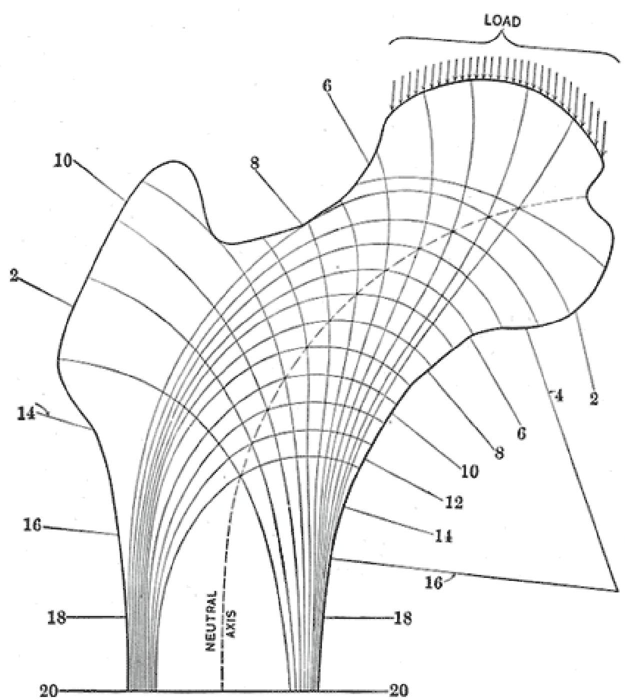


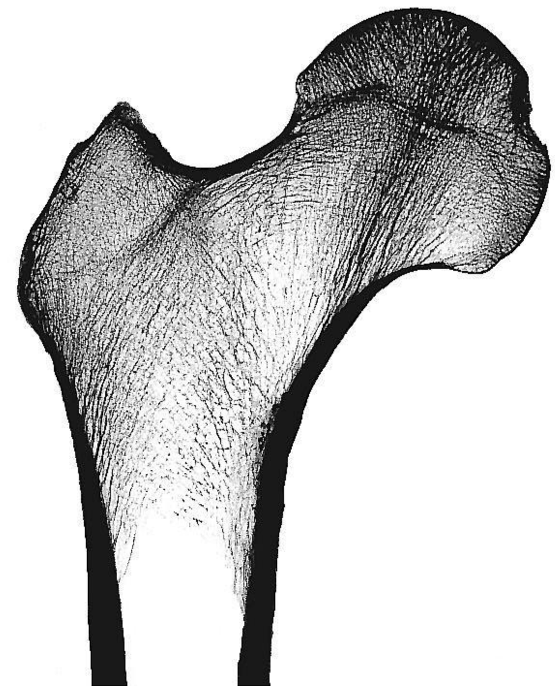
Beer & Johnston (McGraw Hill)

- Principal stresses are the stresses in the planes where the shear stress is zero.

- The highest principal stress is the maximum normal stress while the lowest is the minimum normal stress.

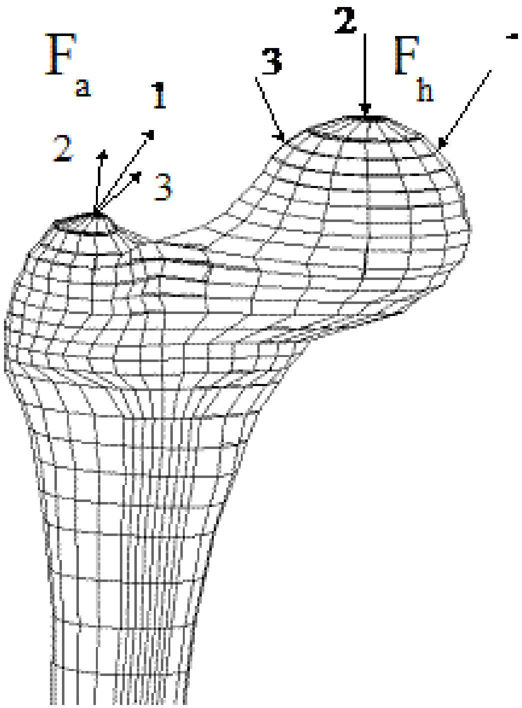
Principal Stresses in the Femur

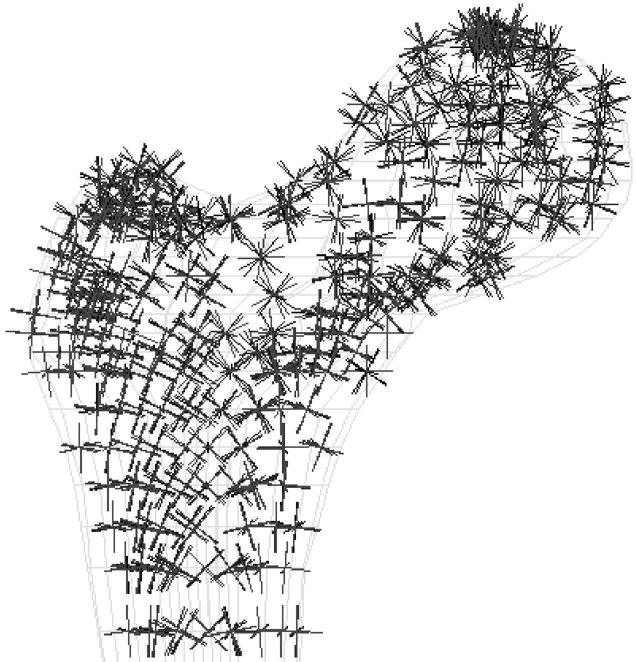


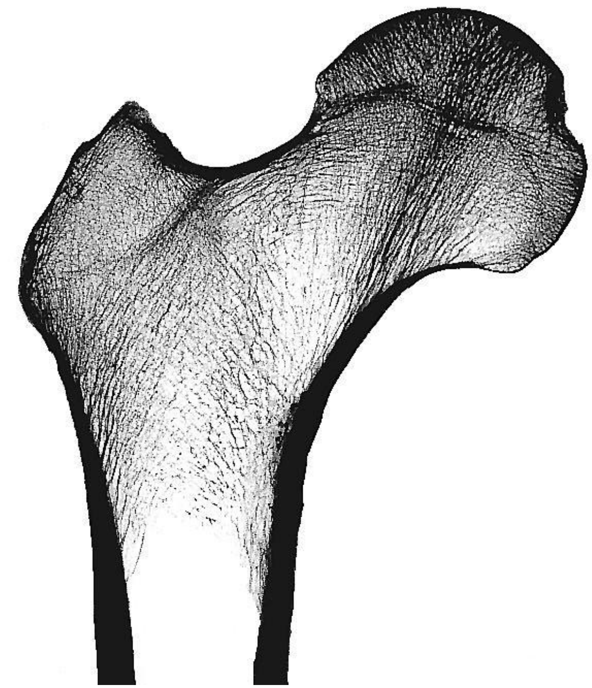


Koch (1917)

Principal Stresses in the Femur







Fernandes, Rodrigues
and Jacobs (1999)

Principal Stresses for a 2D state of stress

Proposed Problem:

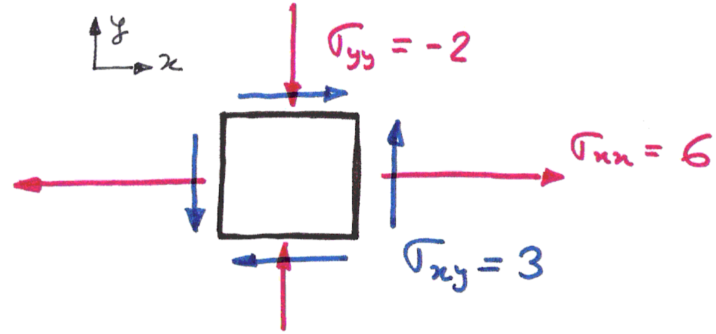
For the given state of plan stress:

$$[\sigma] = \begin{bmatrix} 6 & 3 \\ 3 & -2 \end{bmatrix}$$

Determine the principal stresses and principal directions.

Principal stresses and directions are solution of an eigenvalues and eigenvectors problem:

$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 3 & -2 \end{bmatrix}$$



Principal stresses-eigenvalues

$$\det\left[\sigma - \lambda I\right] = 0 \Rightarrow \det \begin{bmatrix} 6-\lambda & 3 \\ 3 & -2-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (6-\lambda)(-2-\lambda) - 3^2 = 0 \Rightarrow \lambda^2 - 4\lambda - 21 = 0 \Rightarrow \lambda_1 = 7 \; ; \; \lambda_2 = -3$$

$\Rightarrow$ Principal stresses $\boxed{\sigma_1 = 7 \text{ MPa}, \quad \sigma_{\cancel{2}\,3} = -3 \text{ MPa}}$

In the principal reference system (principal directions) the stress state is represented by:

$$\sigma = \begin{bmatrix} 7 & 0 \\ 0 & -3 \end{bmatrix}$$

Principal directions - eigenvectors

$\lambda_1 = 7$ $\quad [\sigma - \lambda I]\{d\} = \{0\} \Rightarrow \begin{bmatrix} 6-7 & 3 \\ 3 & -2-7 \end{bmatrix} \begin{Bmatrix} d_x \\ d_y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$

$\Rightarrow \begin{bmatrix} -1 & 3 \\ 3 & -9 \end{bmatrix} \begin{Bmatrix} d_x \\ d_y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \Rightarrow -d_x + 3d_y = 0$

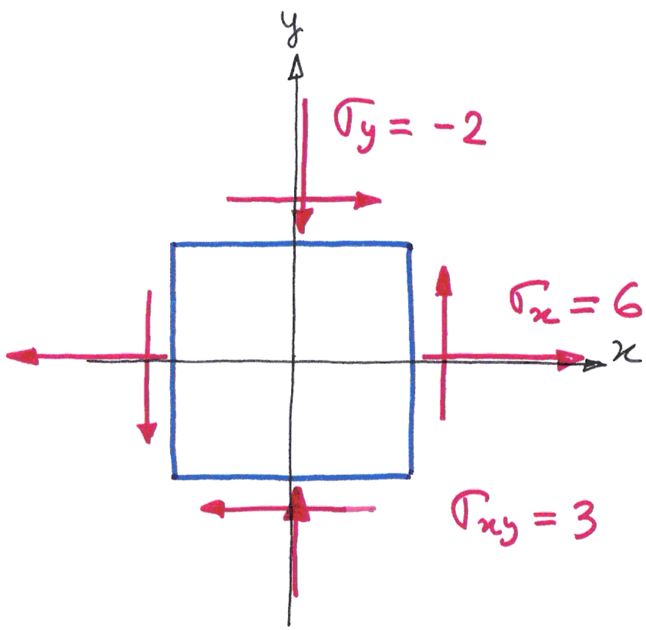
$\Rightarrow \boxed{d_x^1 = 3 d_y^1}$

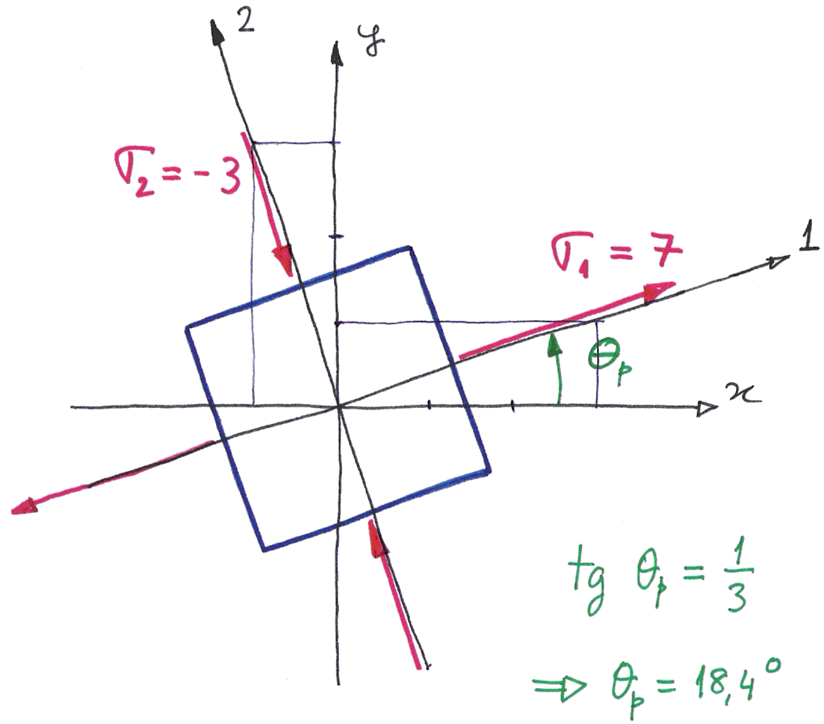
condição que a direcção principal 1 tem de verificar

$\lambda_2 = -3$ $\quad [\sigma - \lambda I]\{d\} = \{0\} \Rightarrow \begin{bmatrix} 6+3 & 3 \\ 3 & -2+3 \end{bmatrix} \begin{Bmatrix} d_x \\ d_y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$

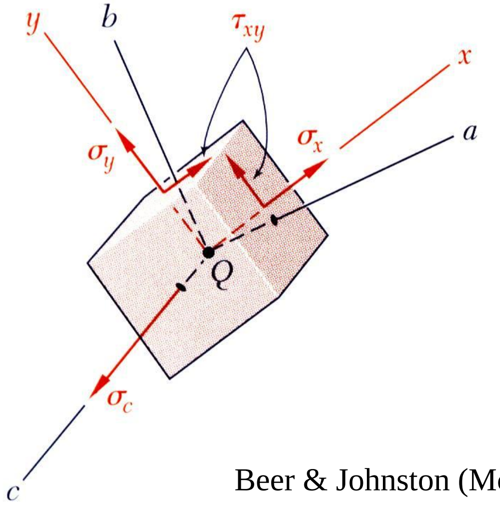
$\Rightarrow \begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix} \begin{Bmatrix} d_x \\ d_y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \Rightarrow 3d_x + d_y = 0$

$\Rightarrow \boxed{d_x^2 = -\frac{1}{3} d_y^2}$

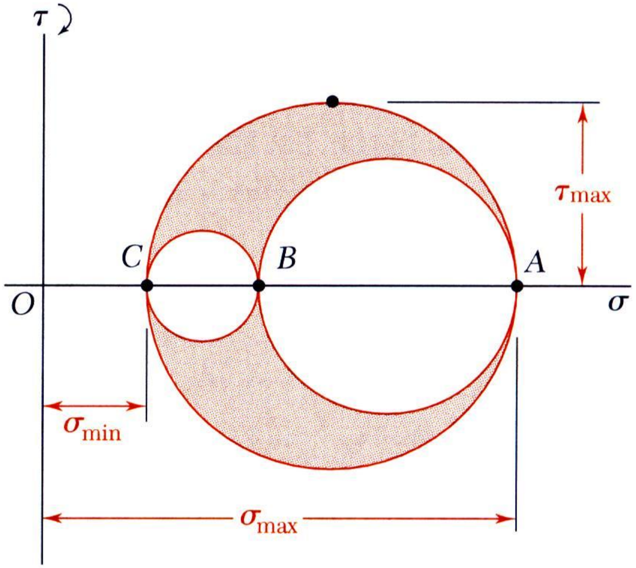

y
$\sigma_y = -2$
$\sigma_x = 6$
x
$\sigma_{xy} = 3$


2
y
$\sigma_2 = -3$
$\sigma_1 = 7$
1
θ_p
x
$tg\ \theta_p = \frac{1}{3}$
$\Rightarrow \theta_p = 18,4^\circ$

Mohr's circle for a 3D state of stress



Beer & Johnston (McGraw Hill)



- Points *A*, *B*, and *C* represent the principal stresses on the principal planes (shearing stress is zero)

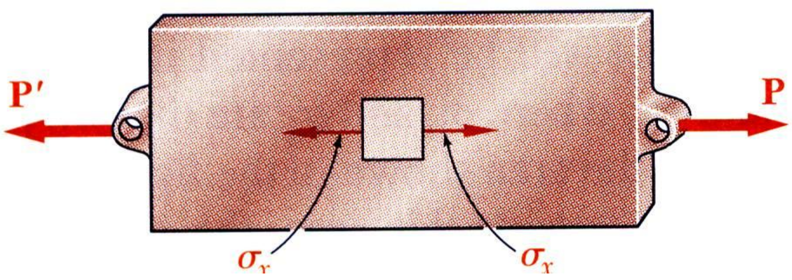
- The three circles represent the normal and shearing stresses for rotation around each principal axis.

- Radius of the largest circle yields the maximum shearing stress.

$$\tau_{max} = \frac{1}{2}\left|\sigma_{max} - \sigma_{min}\right|$$

Failure Criteria

- Failure of a component subjected to uniaxial stress is directly predicted from an equivalent tensile test



Beer & Johnston (McGraw Hill)

- Failure of a component subjected to a general state of stress cannot be directly predicted from the uniaxial state of stress in a tensile test specimen

- Failure criteria are based on the mechanism of failure (ductile vs. brittle materials). Allows comparison of the failure conditions for a uniaxial stress test and biaxial component loading

Ductile Material – Von Mises criterion

Problem:

A cylindrical sample with an outer diameter of d_e=32 mm and a inner diameter of d_i=16 mm, is subject to a bending moment M=140 N.m and a torque of T=210 N.m.

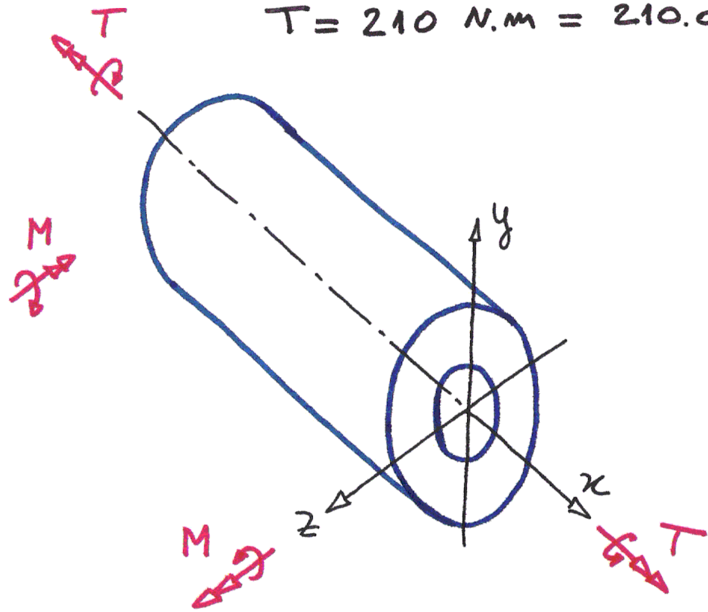
The material is isotropic with a normal yield stress of σ_e=115 MPa.

Verify if under these conditions the material yields.

<u>Loading</u>

$M = 140\ N.m = 140.000\ Nmm$

$T = 210\ N.m = 210.000\ Nmm$



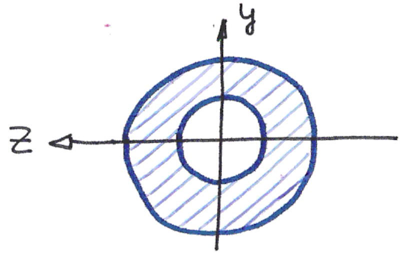
<u>Flexão</u>

$$\sigma = -\frac{My}{I}$$

<u>Torção</u>

$$\tau = \frac{Tr}{J}$$

<u>Cross-section properties</u>



$r_e = 16\ mm$

$r_i = 8\ mm$

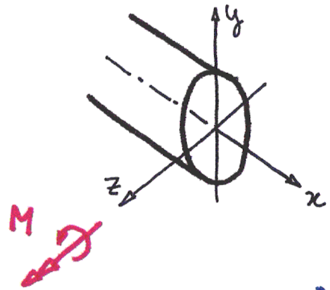
$$I = \frac{\pi}{4}\left(r_e^4 - r_i^4\right) = \frac{\pi}{4}\left(16^4 - 8^4\right) = 48{,}25 \times 10^3\ mm^4$$

$$\Rightarrow I = 48{,}25 \times 10^3\ mm^4$$

$$J = \frac{\pi}{2}\left(r_e^4 - r_i^4\right) = \frac{\pi}{2}\left(16^4 - 8^4\right) = 96{,}5 \times 10^3\ mm^4$$

Bending

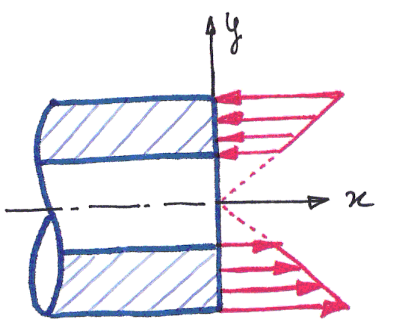
$$\sigma = -\frac{M y}{I}$$



Tensão máxima (em módulo) para os valores extremos de y, $|y_{max}| = r_e$

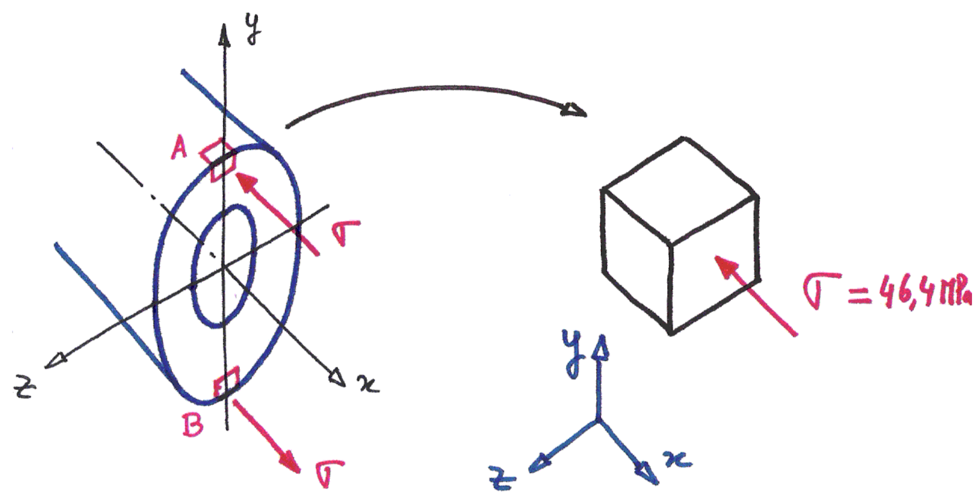
$$\sigma_{max} = \frac{M r_e}{I} = \frac{140\times10^{3} \times 16}{48{,}25\times10^{3}} = 46{,}4\ MPa \Rightarrow \boxed{\sigma_{max} = 46{,}4\ MPa}$$

Normal stress function of y

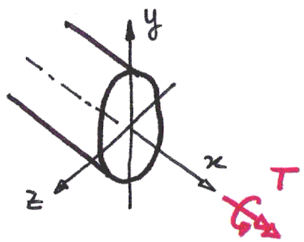


$\sigma_{max} = 46{,}4\ MPa$

State of stress where the bending stress is maximum:



Torsion

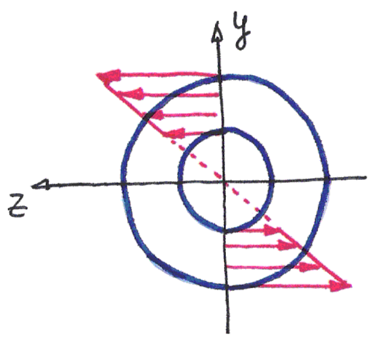


$$\tau = \frac{T r}{J}$$

Tensão máxima para os valores extremos de r, $r_{max} = r_e$

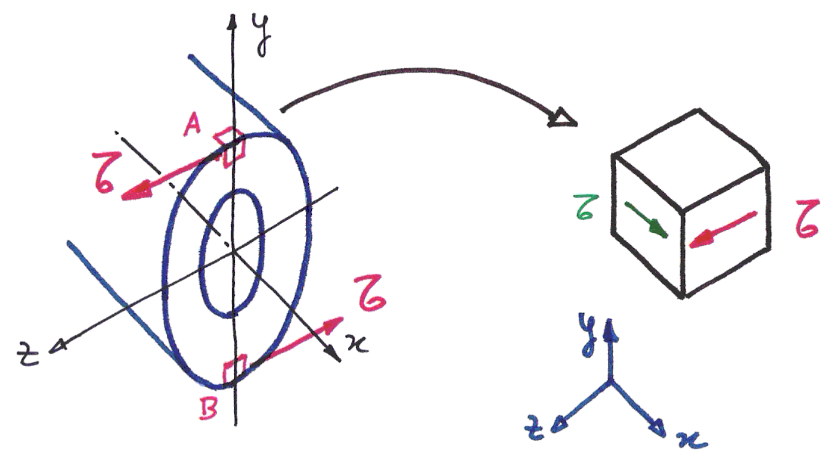
$$\tau_{max} = \frac{T r_e}{J} = \frac{210 \times 10^3 \times 16}{96{,}5 \times 10^3} = 34{,}8\ \text{MPa} \Rightarrow \boxed{\tau_{max} = 34{,}8\ \text{MPa}}$$

Shear stress function of r



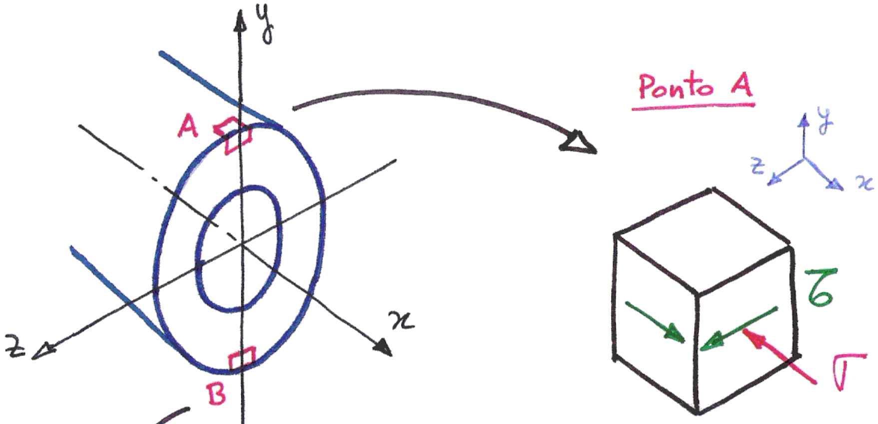
$\tau_{max} = 34{,}8$ MPa

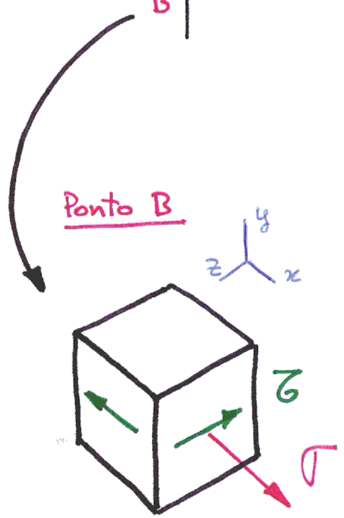
State of stress where the shear is maximum:



Bending + Torsion (combined where both shear and normal stress have the maximum values)

$y = \pm 16 \text{ mm}$





$$[\sigma] = \begin{array}{c c} & \begin{array}{ccc} x & y & z \end{array} \\ \begin{array}{c} x \\ y \\ z \end{array} & \begin{bmatrix} \sigma_x & 0 & \tau_{xz} \\ 0 & 0 & 0 \\ \tau_{xz} & 0 & 0 \end{bmatrix} \end{array} \longrightarrow [\sigma] = \begin{bmatrix} \sigma_x & \tau_{xz} \\ \tau_{xz} & 0 \end{bmatrix}$$

2D state of stress (plane stress)

State of stress at A

$(y = +16 \text{ mm})$

$$\sigma_A = \begin{bmatrix} \sigma_x & \tau_{xz} \\ \tau_{xz} & 0 \end{bmatrix} = \begin{bmatrix} -46,4 & +34,8 \\ +34,8 & 0 \end{bmatrix}$$

State of stress at B

$(y = -16 \text{ mm})$

$$\sigma_B = \begin{bmatrix} \sigma_x & \tau_{xz} \\ \tau_{xz} & 0 \end{bmatrix} = \begin{bmatrix} +46,4 & -34,8 \\ -34,8 & 0 \end{bmatrix}$$

Yield Criterion -Von Mises criterion (Ductile Materials)

• Yield occurs when the distortion energy per unit volume is greater than that occurring in a tensile test specimen at yield.

• In the distortion energy is possible to identify a term to compare directile to the yied normal stress given by the tensile test.

For a plane stress state (σ_a and σ_b are the principal stresses):

$$u_d < u_Y$$

$$\frac{1}{6G}\left(\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2\right) < \frac{1}{6G}\left(\sigma_Y^2 - \sigma_Y \times 0 + 0^2\right)$$

$$\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2 < \sigma_Y^2$$

• This term is the equivalent **Von Mises Stress**

Yield Criterion -Von Mises criterion (Ductile Materials)

• In pratice we compare the Von Mises stress with the yield stress of the material.

Thus, the failure (yield) occurs when:

$$\frac{\sigma_{VM}}{\sigma_e} > 1$$

σ_e = Yield stress of the material.

$$\boxed{\sigma_{VM} = \frac{1}{\sqrt{2}}\left[(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{xz}^2\right)\right]^{1/2}}$$

Von Mises Stress

For the proposed problem

$$\sigma_y = 0 \, , \quad \sigma_z = 0 \, , \quad \tau_{xy} = 0 \, , \quad \tau_{yz} = 0$$

$$\sigma_{VM} = \frac{1}{\sqrt{2}} \left[\sigma_x^2 + \sigma_x^2 + 6\,\tau_{xz}^2 \right]^{1/2} = \frac{1}{\sqrt{2}} \left[2 \times 46{,}4^2 + 6 \times 34{,}8^2 \right]^{1/2}$$

$$\Longrightarrow \boxed{\sigma_{VM} = 76{,}1 \text{ MPa}}$$

Comparing with the yield stress given for the material $(\sigma_e = 115 \text{ MPa})$

$$\frac{\sigma_{VM}}{\sigma_e} = \frac{76{,}1}{115} = 0{,}66 < 1$$

The sample is safe. Yield does not occur.

Brittle Material – Mohr criterion

Problem:

A cylindrical sample with an outer diameter of d_e=32 mm and a inner diameter of d_i=16 mm, is subject to a bending moment M=140 N.m and a torque of T=210 N.m.

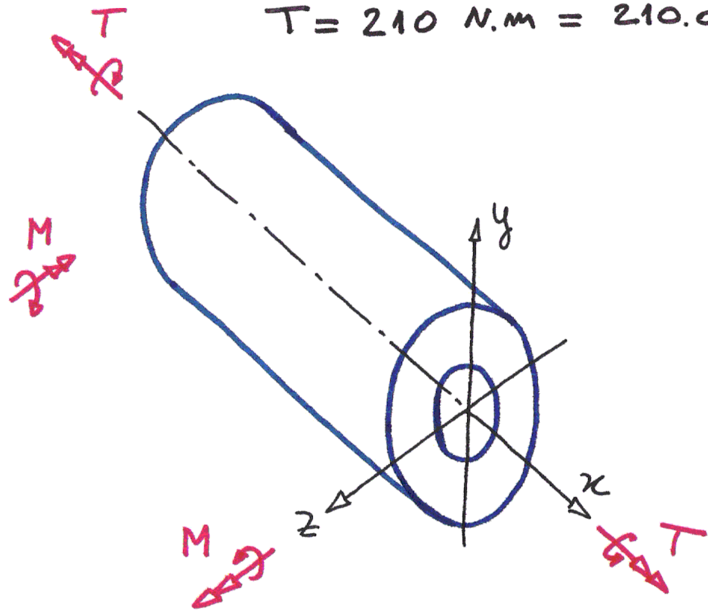
The material is isotropic and brittle with failure tensile stress of σ_{tf}=133 MPa and failure compressive stress of σ_{cf}=195 Mpa.

Verify if under these conditions the material fails.

Loading

$M = 140\ N.m = 140.000\ Nmm$

$T = 210\ N.m = 210.000\ Nmm$



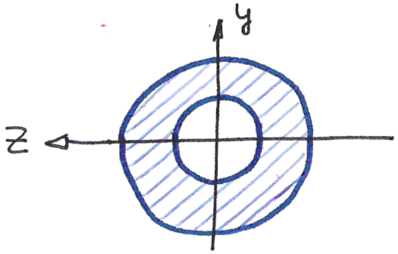
Flexão

$$\sigma = -\frac{My}{I}$$

Torção

$$\tau = \frac{Tr}{J}$$

Cross-section properties



$r_e = 16\ mm$

$r_i = 8\ mm$

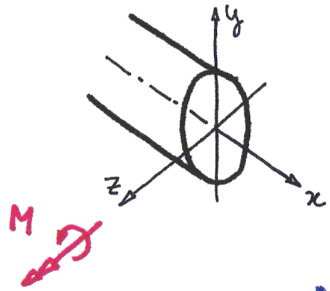
$$I = \frac{\pi}{4}\left(r_e^4 - r_i^4\right) = \frac{\pi}{4}\left(16^4 - 8^4\right) = 48{,}25 \times 10^3\ mm^4$$

$$\Rightarrow I = 48{,}25 \times 10^3\ mm^4$$

$$J = \frac{\pi}{2}\left(r_e^4 - r_i^4\right) = \frac{\pi}{2}\left(16^4 - 8^4\right) = 96{,}5 \times 10^3\ mm^4$$

Bending

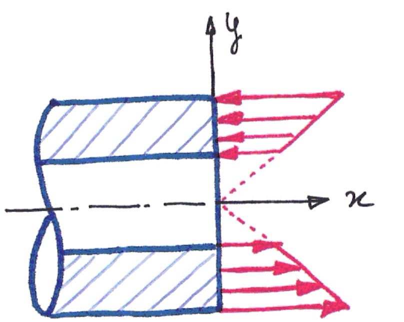
$$\sigma = -\frac{M\,y}{I}$$



Tensão máxima (em módulo) para os valores extremos de y, $|y_{max}| = r_e$

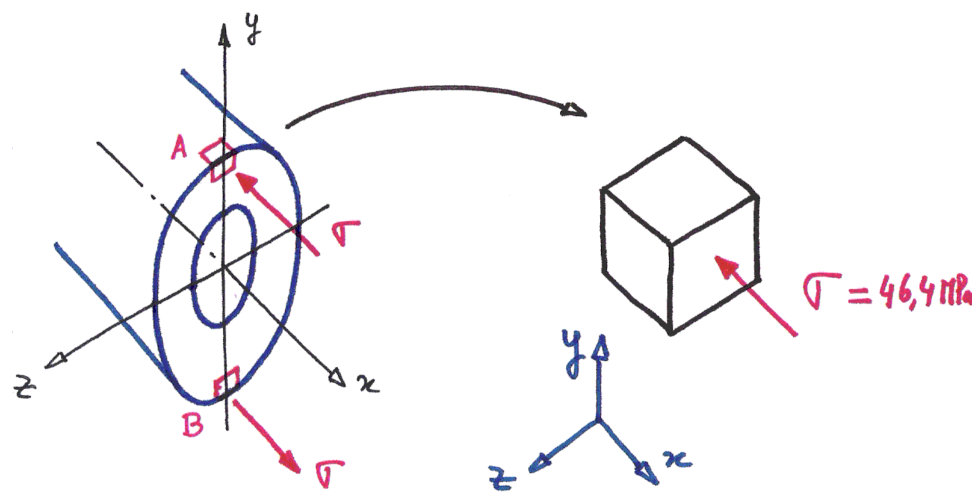
$$\sigma_{max} = \frac{M r_e}{I} = \frac{140\times10^3 \times 16}{48{,}25\times10^3} = 46{,}4 \text{ MPa} \Rightarrow \boxed{\sigma_{max} = 46{,}4 \text{ MPa}}$$

Normal stress function of y

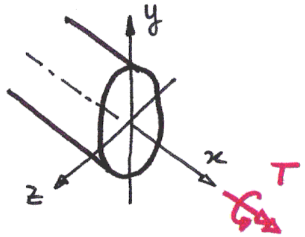


$\sigma_{max} = 46{,}4$ MPa

State of stress where the bending stress is maximum:



Torsion

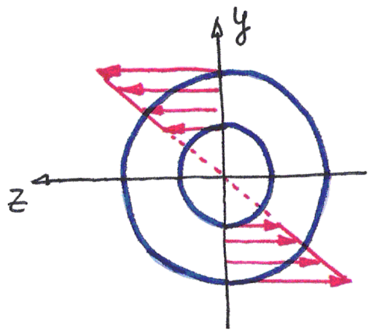


$$\tau = \frac{T r}{J}$$

Tensão máxima para os valores extremos de r, $r_{max} = r_e$

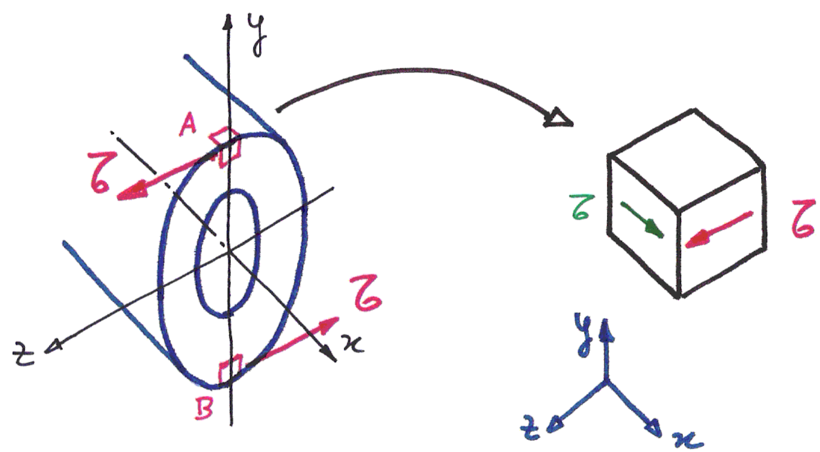
$$\tau_{max} = \frac{T r_e}{J} = \frac{210 \times 10^3 \times 16}{96{,}5 \times 10^3} = 34{,}8 \text{ MPa} \Rightarrow \boxed{\tau_{max} = 34{,}8 \text{ MPa}}$$

Shear stress function of r



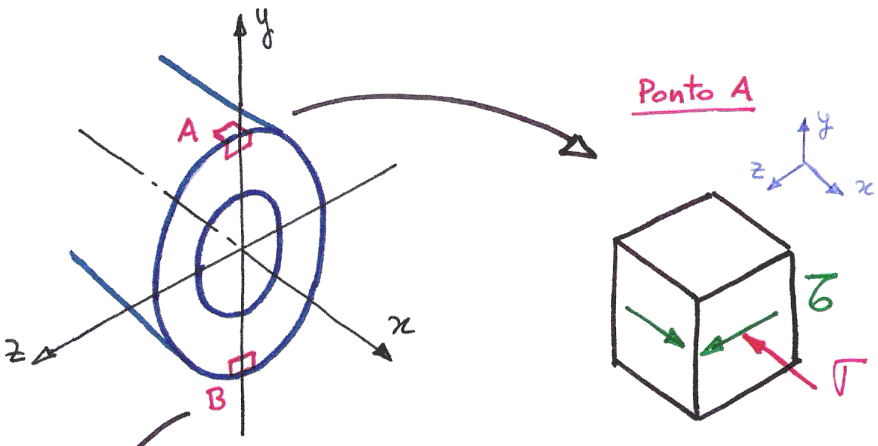
$\tau_{max} = 34{,}8$ MPa

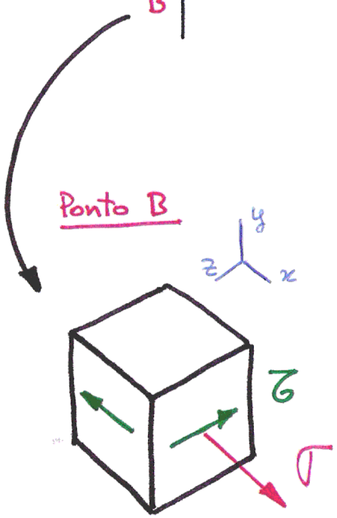
State of stress where the shear is maximum:



Bending + Torsion (combined where both shear and normal stress have the maximum values)

$y = \pm 16 \text{ mm}$





$$[\sigma] = \begin{array}{c} \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} \sigma_x & 0 & \tau_{xz} \\ 0 & 0 & 0 \\ \tau_{xz} & 0 & 0 \end{bmatrix} \end{array}$$

2D state of stress (plane stress)

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xz} \\ \tau_{xz} & 0 \end{bmatrix}$$

State of stress at A

$(y = +16\ \text{mm})$

$$\sigma_A = \begin{bmatrix} \sigma_x & \tau_{xz} \\ \tau_{xz} & 0 \end{bmatrix} = \begin{bmatrix} -46,4 & +34,8 \\ +34,8 & 0 \end{bmatrix}$$

State of stress at B

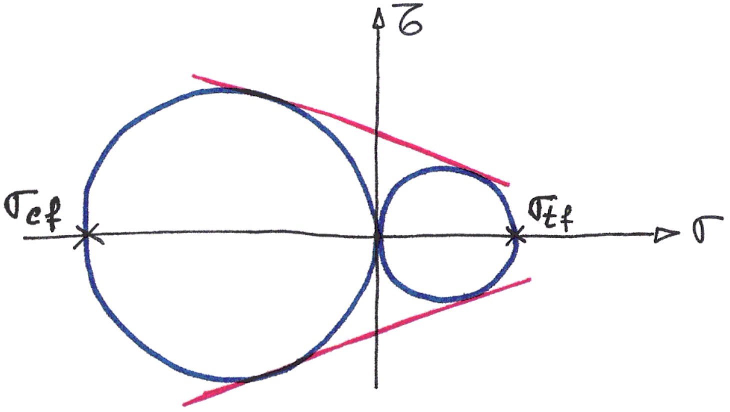
$(y = -16\ \text{mm})$

$$\sigma_B = \begin{bmatrix} \sigma_x & \tau_{xz} \\ \tau_{xz} & 0 \end{bmatrix} = \begin{bmatrix} +46,4 & -34,8 \\ -34,8 & 0 \end{bmatrix}$$

Brittle Material – Mohr criterion

failure occurs when

$$\boxed{\frac{\sigma_1}{\sigma_{tf}} - \frac{\sigma_3}{\sigma_{cf}} > 1}$$



where σ_1, σ_3 are the principal stresses (the highest and the lowest)

σ_{tf} is the limiting tensile stress (tensile test)

σ_{cf} is the limiting compressive stress (compression test)

σ_1 e σ_3 Can be positive or negative

Principal stresses for the proposed problem

Ponto A

$(y = +16\,mm)$

$$\sigma_A = \begin{bmatrix} -46{,}4 & 34{,}8 \\ 34{,}8 & 0 \end{bmatrix}$$

$$\det\left[\sigma - \lambda I\right] = 0 \Rightarrow \det\begin{bmatrix} -46{,}4-\lambda & 34{,}8 \\ 34{,}8 & -\lambda \end{bmatrix} = 0$$

$$\Rightarrow (-46{,}4-\lambda)(-\lambda) - 34{,}8^2 = 0 \Rightarrow \lambda^2 + 46{,}4\,\lambda - 34{,}8^2 = 0$$

$$\Rightarrow \underset{\sigma_1}{\underset{\shortparallel}{\lambda_1}} = +18{,}6\ MPa \ ; \ \underset{\sigma_3}{\underset{\shortparallel}{\lambda_2}} = -65{,}0\ MPa$$

$$\Rightarrow \boxed{\sigma_1 = 18{,}6\ MPa \ ; \ \sigma_3 = -65{,}0\ MPa}$$

Ponto B

($y = -16$ mm)

$$\sigma_B = \begin{bmatrix} +46,4 & -34,8 \\ -34,8 & 0 \end{bmatrix}$$

$$\det\left[\sigma - \lambda I\right] = 0 \quad \Rightarrow \quad \det\begin{bmatrix} 46,4-\lambda & -34,8 \\ -34,8 & -\lambda \end{bmatrix} = 0$$

$$\Rightarrow (46,4-\lambda)(-\lambda) - 34,8^2 = 0 \quad \Rightarrow \quad \lambda^2 - 46,4\,\lambda - 34,8^2 = 0$$

$$\Rightarrow \underset{\sigma_1}{\underset{\parallel}{\lambda_1}} = 65,0 \text{ MPa} \quad ; \quad \underset{\sigma_3}{\underset{\parallel}{\lambda_2}} = -18,6 \text{ MPa}$$

$$\Rightarrow \boxed{\sigma_1 = 65,0 \text{ MPa} \quad ; \quad \sigma_3 = -18,6 \text{ MPa}}$$

Mohr's criterion – Point A

$\sigma_1 = 18{,}6\ ;\quad \sigma_2 = 0\ ;\quad \sigma_3 = -65{,}0\ \text{MPa} \qquad \sigma_{tf} = 133\ \text{MPa}\ ;\quad \sigma_{cf} = 195\ \text{MPa}$

$$\frac{\sigma_1}{\sigma_{tf}} - \frac{\sigma_3}{\sigma_{cf}} = \frac{18{,}6}{133} - \frac{-65{,}0}{195} = \frac{18{,}6}{133} + \frac{65{,}0}{195} = 0{,}47 < 1$$

Thus, there is no fail

Mohr's criterion – Point B

$\sigma_1 = 65{,}0;\quad \sigma_2 = 0\ ;\quad \sigma_3 = -18{,}6\ \text{MPa} \qquad \sigma_{tf} = 133\ \text{MPa}\ ;\quad \sigma_{cf} = 195\ \text{MPa}$

$$\frac{\sigma_1}{\sigma_{tf}} - \frac{\sigma_3}{\sigma_{cf}} = \frac{65{,}0}{133} - \frac{-18{,}6}{195} = \frac{65{,}0}{133} + \frac{18{,}6}{195} = 0{,}58 < 1$$

Thus, there is no fail

Remark: At B the risk of the material is bigger (0.58 > 0.47) because the sample is in tension due to bending and the limiting stress in tension is smaller than in compression.

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