Bone Tissue Mechanics

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PART 1

Introduction

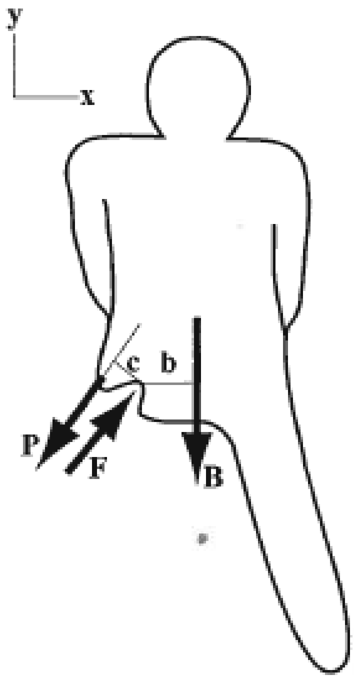
- The objective of this course is to study basic concepts on hard tissue mechanics.
- **Hard tissue** is the structural material of the **skeleton**, mainly bone and cartilage. In this course the focus will be on **bone biomechanics**.
- **The skeleton** is a mechanical organ. Its primary functions are to transmit forces from one part of the body to another and protect certain organs from mechanical forces that could damage them.

Introduction

To study the effect of loads on the skeleton, and in particular in bone we have to know:

- ➢ Which loads are applied to bone?
  - •Basically loads are transmitted by joint, so the question is how to know the forces in joints.
  - •It is possible to obtain an order of magnitude of this loads using free body diagrams and static analysis.
- ➢ What is the effect of these load in bones?
  - • Concept of mechanical stress and strain. Bone as a deformable bone.
- ➢ How bone support these loads?
  - •Bone as a structural material.
  - •Mechanical properties of Bone
  - •Bone adaptation to mechanical loads.

Forces in the Hip Joint



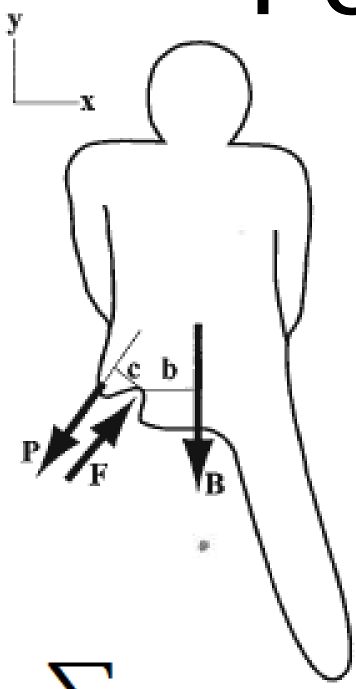
Modelling assumptions:

- "single leg stance phase" of gait.
- two-dimensional analysis.

P – Abductor muscles; F – Joint reaction force acting in the middle of the acetabulum.; B – weight of the body on the leg. W – Body weight.

Because each lower member is about (1/6)W, B=(5/6)W

Forces in the Hip Joint



$$\sum M = 0 \qquad cP - bB = 0 \qquad P = \frac{b}{c}B = \frac{b}{c} \times \frac{5}{6}W$$

The lengths b and c can be estimated from X-ray. It was found that: $2 < \frac{b}{c} < 3.5$

Assuming $\frac{b}{c} = 2.4 \Rightarrow \quad P = 2.4 \times \frac{5}{6}W \Rightarrow \boxed{P = 2W}$

$$\sum F_x = 0 \qquad F_x - P_x = 0 \Leftrightarrow F_x - 2W\sin\theta = 0$$

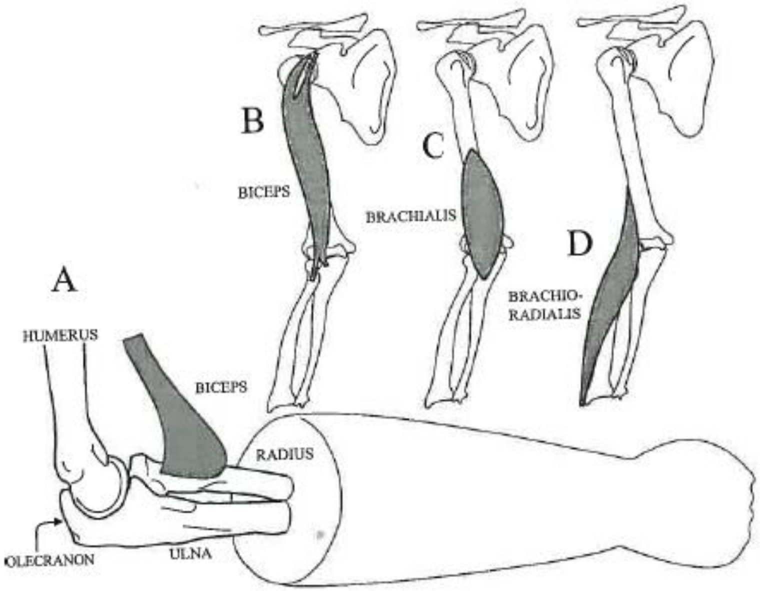
$$\sum F_y = 0 \qquad F_y - P_y - B = 0 \Leftrightarrow F_y - 2W\cos\theta - \frac{5}{6}W = 0$$

θ is the angle between the abductor muscle line and the y-axis.

Assuming $\theta = 30^o \qquad F_x = W \qquad F_y = \left(2\cos 30 + \frac{5}{6}\right)W = 2.57W$

Remark: The ratio b/c is critical for the hip load magnitude.

Forces in the Elbow Joint

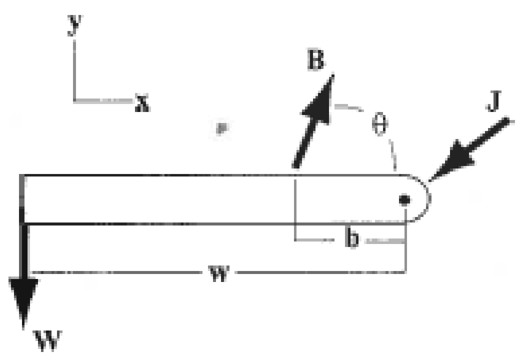


W – Weight in the hand; J – reaction in the joint; B – biceps (and brachial) force

$$\sum M = 0 \qquad wW - bB\sin\theta = 0$$

$$\sum F_x = 0 \qquad B\cos\theta - J_x = 0$$

$$\sum F_y = 0 \qquad B\sin\theta - W - J_y = 0$$



$$B = \frac{a}{b} W \sin\theta \qquad J_x = B\cos\theta \quad J_y = B\sin\theta - W$$

If θ=75º; w = 0.35 m and b = 0.04 m thus:

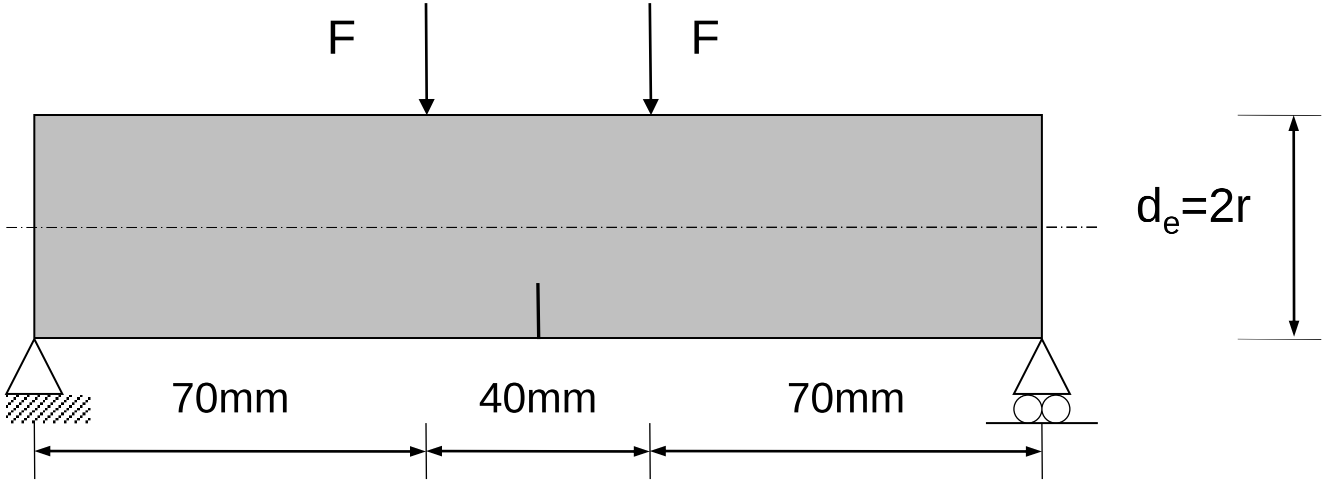
$$B = 9.1W;\ \ J_x = 2.3W \ \ and\ \ J_y = 7.8W$$

$$|J| = \sqrt{J_x^2 + J_{xy}^2} = 8.1W$$ and orientation is: $$arctan\left(\frac{J_y}{J_x}\right) = 74^o$$

Stress in bending

Problem (4 point bending)

A bone sample, with outer diameter d_e=32 mm and inner diameter d_i=16 mm, is subject to a four-point test (see figure, F=1 KN).Determine the maximum bending normal stress .

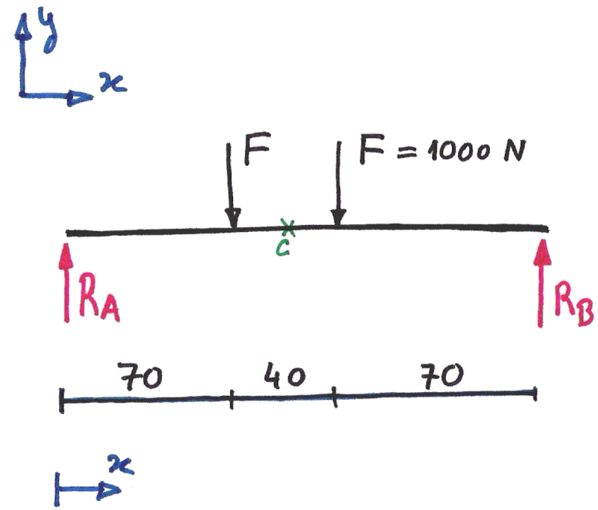


Note: $1\,\frac{N}{mm^2} = \frac{1\,N}{1\,mm^2} = \frac{1\,N}{(10^{-3}\,m)^2} = \frac{1\,N}{10^{-6}\,m^2} = 10^6\,\frac{N}{m^2} = 10^6\,Pa$

ou seja $\boxed{1\,\frac{N}{mm^2} = 1\,MPa}$

Solution:

1) Reactions:



Static equilibrium:

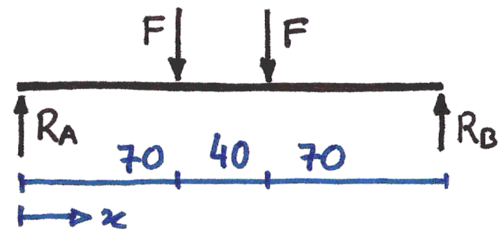
$\Sigma F_y = 0 \Rightarrow R_A + R_B = 2F$

$\Sigma M_c = 0 \Rightarrow R_A = R_B$

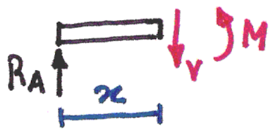
$\boxed{R_A = R_B = F}$

$\rightarrow \boxed{R_A = R_B = 1000\,N}$

2) Shear and Bending Moment diagram:



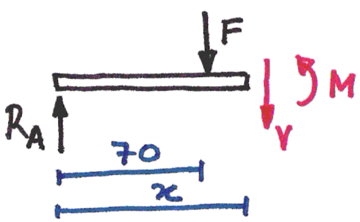
$0 < x < 70$



$\sum F_y = 0 \Rightarrow R_A - V = 0 \Rightarrow V = R_A \Rightarrow \boxed{V = 1000\ N}$

$\sum M = 0 \Rightarrow -R_A x + M = 0 \Rightarrow M = R_A x \Rightarrow \boxed{M = 1000 \times x \quad N.mm}$

$70 < x < 110$

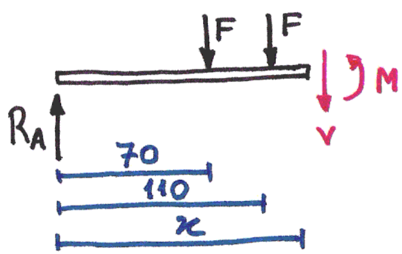


$\sum F_y = 0 \Rightarrow R_A - F - V = 0 \Rightarrow V = R_A - F = 0 \Rightarrow \boxed{V = 0}$

$\sum M = 0 \Rightarrow -R_A x + F(x - 70) + M = 0$

$\Rightarrow M = \cancel{R_A x} - \cancel{F x} + F \times 70 \Rightarrow \boxed{M = 70.000\ N.mm}$

$110 < x < 180$

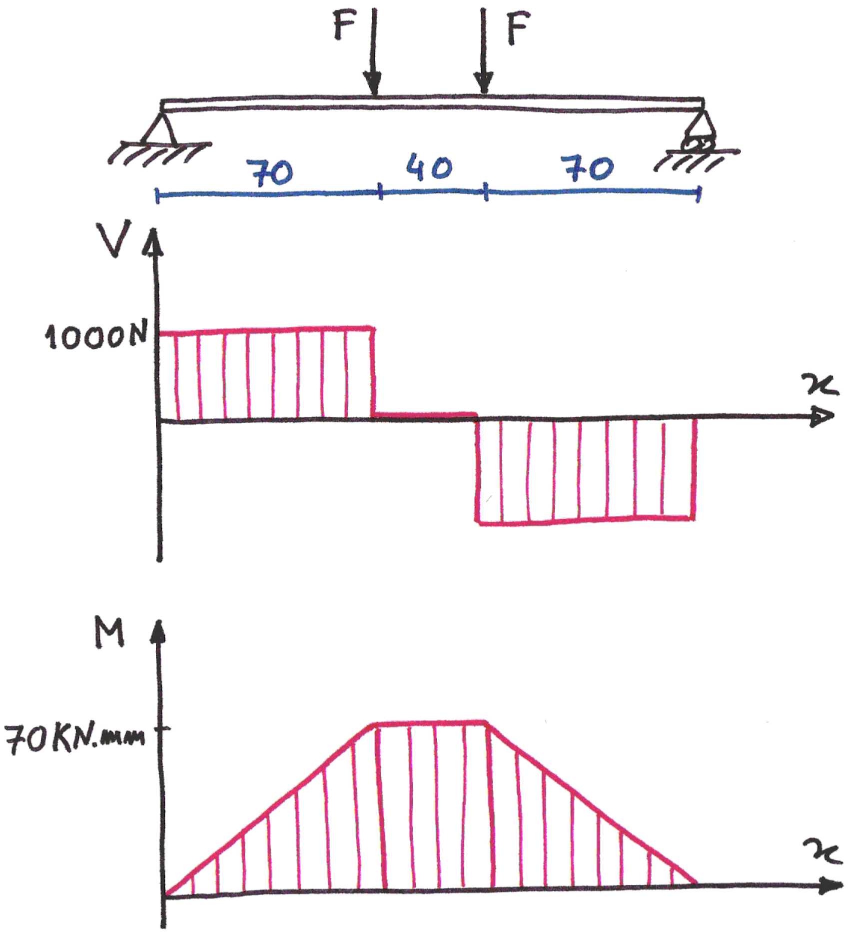


$\sum F_y = 0 \Rightarrow R_A - 2F - V = 0 \Rightarrow V = R_A - 2F = -F \Rightarrow \boxed{V = -1000\ N}$

$\sum M = 0 \Rightarrow -R_A x + F(x - 70) + F(x - 110) + M = 0$

$\Rightarrow M = \cancel{R_A x} - \cancel{F x} + F \times 70 - F x + F \times 110$

$\Rightarrow M = F \times 180 - F x \Rightarrow \boxed{M = 180.000 - 1000 \times x \quad N.mm}$



Note: for $70 < x < 110$

the sample is subject to pure bending

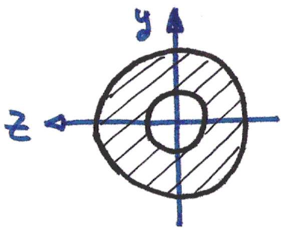
3. Bending normal stress

$$\longrightarrow \sigma = -\frac{M\,y}{I}$$

The maximum stress occurs in the section where the absolute value of the bending moment is maximum at the points where the distance to the neutral axis is maximum.

$$|y_{max}| = r_e \qquad \sigma_{max} = \frac{M_{max}\, r_e}{I}$$

Moment of inertia



$r_e = 16$ mm

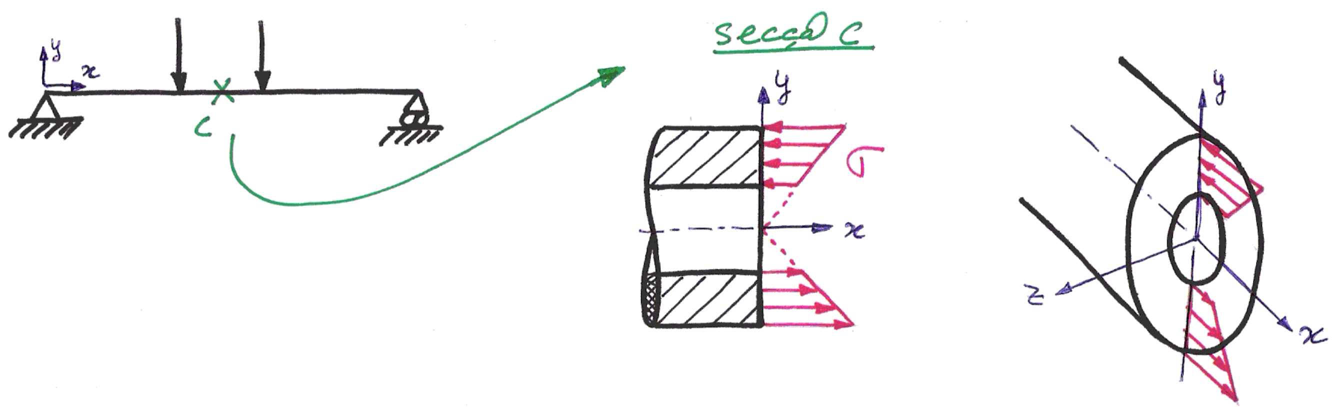
$r_i = 8$ mm

$$I = \frac{\pi}{4}\left(r_e^4 - r_i^4\right) = \frac{\pi}{4}\left(16^4 - 8^4\right) = 48{,}25\times10^3\ mm^4$$

$$I = 48{,}25\times10^3\ mm^4$$

maximum stress

$$\sigma_{max} = \frac{M_{max}\, r_e}{I} = \frac{70\times10^3 \times 16}{48{,}25\times10^3} = 23{,}2\ MPa \longrightarrow \boxed{\sigma_{max} = 23{,}2\ MPa}$$



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